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ABSTRACT

This volume reports research conducted to provide the National Science Foundation (NSF) with information concerning the existing range of beliefs and opinions about the impact of the hand-held calculator on pre-college educational practice. A literature search and several surveys of groups of individuals involved in calculator manufacture and sales or in education were conducted. In addition, four position papers by prominent educators were obtained. The body of the report presents summaries of information collected relevant to availability of calculators, arguments for and against their use in schools, ways in which they are now used in schools, and research findings. Five recommendations related to study of and preparation for implementation of calculator use are made. Appendices to the report present an annotated list of references, the detailed findings of the surveys, and the position papers: Immerzeel, Ockenga, and Tarr discuss plausible instances with which to use calculators; Pollak proposes criteria for redesigning the curriculum; Weaver makes suggestions for needed research; and Usiskin and Bell provide some perspectives on curriculum revision. (SD)

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Electronic Hand Calculators:

The Implications for Pre-College Education

Final Report

prepared for

The National Science Foundation

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February 1976

Project Director: Marilyn N. Suydam, The Ohio State University

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SE 021 321

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Foreword

The information for this report was largely collected in May, June, and July 1975; the Interim Report (originally intended to be the Final Report) was prepared in August 1975. The present paper is an extension, based on the original data and an analysis of what has occurred since the Interim Report was prepared, but without further data collection.

Richard K. Shumway participated in the project and was responsible for developing significant portions of the Interim Report. Because the Project Ending Date was extended without further funding, other professional commitments kept him from participating in the preparation of this Final Report, and he requested that his name not be used in a co-author sense. Nevertheless, many sections of this Report are directly attributable to his earlier analysis: I have attempted to indicate these in the Report. I would also like to express my appreciation for his help.

Marilyn N. Suydam
The Ohio State University

Appreciation is also extended to others who helped in the preparation of the Interim Report: Suzanne Damarin and Paul Wozniak, doctoral students at The Ohio State University, for their assistance in collecting and collating information; Louraine Wagner and Beverly Brooks, for typing project materials and managing certain aspects of the project; colleagues at the University and personnel at NSF for their helpful suggestions -- and most of all, to all those throughout the country who so willingly shared information.

Electronic Hand Calculators:
The Implications for Pre-College Education

I. Introduction

Explanation of the Study

There have been many misconceptions about this project. Therefore, it seems appropriate that this report should begin by stating three things that this project was not designed to do:

- It was not designed to compare the reactions or beliefs of different groups about hand-held* calculators.
- It was not designed to collect specific uses of calculators in the classroom.
- It was not designed to be the precursor of a development project (much as that may be needed).

It was designed to provide a report to The National Science Foundation on the range of beliefs and reactions about calculators, and in particular on the arguments that were being used to support positions strongly favorable and strongly negative toward the use of calculators in elementary and secondary schools. From various sources, an analysis of the status of the calculator was to be developed.

In connection with this:

(1) A restricted questionnaire survey was conducted: the sampling was by design nationwide, but it was not random. Therefore, the results cannot be statistically compared for the purpose of

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* Hereafter, the word calculator will be used to refer to the hand-held calculator, also termed the electronic calculator, the mini calculcutor (not in the computer sense), or the pocket calculator.

generalizing. The questions were designed simply to elicit the range of reactions, to aid in identifying the arguments being used.

(2) The literature was surveyed to identify what was being said in writing to argue for or against using calculators.

(3) Manufacturers and distributors were surveyed in an attempt to secure information on the current and future status of sales and development.

(4) Experiential reports and research reports were scrutinized to ascertain what has already been learned about how to use calculators and the effect of using calculators.

(5) Telephoning to follow up on information, attending meetings and workshops (local, state, regional, and national) to learn what was said by speakers and audiences, talking with teachers, and similar activities were also part of the process of securing information.

(6) Position papers were requested on several topics, to provide in-depth, thoughtful statements about various aspects related to the use of calculators.

Thus, the original intent of this project was simply:

(1) To collect information regarding the use or non-use of calculators, and to form a list of the reasons why educators and others believed that the calculator should be used in schools or why the calculator should be banned in schools;

(2) To analyze the arguments reported by those questioned and in the literature, in order to determine the potential impact or lack of impact of the calculator on the curriculum; and

(3) To develop a critical analysis of what has and has not been

done with calculators at pre-college levels, what knowledge is or is not available about them, and what implications this has for education at the pre-college level.

Procedures Used in the Study: Further Explication

The steps taken to meet the purposes of the study are described in more detail in this section.

(1) Collect information

Information was collected in three ways: by means of questionnaires; by searching the literature; and by attending meetings and conferences.

(a) Questionnaires

Nine target audiences were identified: calculator manufacturers; supply companies selling calculators; other marketing outlets; state supervisors of mathematics; school districts using and/or studying the use of calculators, not using calculators, or banning the use of calculators; mathematics teacher educators; decision-makers in both elementary and secondary schools (including teachers, administrators, and supervisors); publishers of elementary- and secondary-school textbooks; and curriculum developers.

As the questionnaires to be sent to these groups were being prepared, it became evident that these audiences were not all distinct in their composition. Therefore, five questionnaires were developed, to be sent to: calculator manufacturers and distributors, state supervisors of mathematics, school personnel, teacher educators, and textbook publishers. (These questionnaires are contained in Appendix B.)

Certain questions or points were duplicated on two or more questionnaires, but in general the questions asked of one group differed from those asked another group. The intent was to secure a range of "pro" and "con" answers, rather than to compare the responses of one group with the responses of another. In some cases, telephone contacts were made to secure basic or additional information; in most cases, the questionnaires were sent and returned by mail.

The samples were identified in various ways:

- Calculator manufacturers and distributors were determined primarily by perusal of the literature and advertisements.

All those identified (N = 39) were sent a questionnaire; responses were received from only 7. In an attempt to secure further information, some were contacted by telephone. In addition, a "blind" request for advertising information was sent, and information on specifications of various calculators was collated from these materials.

- Marketing outlets in 20 selected cities were identified and a sample contacted by telephone. However, the sampled outlets indicated that they were unable to provide the type of information requested. Some similar information was, however, secured from marketing journals.

- A list of all those responsible for supervising mathematics in state departments of education was secured, and each person was sent a questionnaire (N = 86, plus 13 Canadian supervisors). Responses were received from 65 persons in 33 states and several provinces.

- School districts and personnel involved in using calculators or taking a position on banning calculators were identified by such procedures as scanning news reports, contacting state supervisors, and querying teacher educators, in addition to contacts made through meetings and conferences. Each person identified was sent a questionnaire (N = 58). Responses were received from 16 teachers and 16 other school personnel, in 20 states.

- Teacher educators who were in a position to make substantive statements about the implications of using calculators were identified primarily through published membership lists. An attempt was made to send a questionnaire to at least one person in each state, as well as to all major curriculum developers. Of 87 questionnaires sent, 78 were returned, from 39 states.

- All publishers of elementary- and secondary-school mathematics textbooks (N = 26) were sent a questionnaire. Responses were received from 13.

(b) Literature

Literature from the following sources was compiled and reviewed:

- Reports from calculator manufacturers and distributors.

- Curriculum materials in ERIC and from other sources.

- Other reports in ERIC (e.g., opinion papers).

- Newspapers.

- Educational journals listed in ERIC's Current Index to Journals in Education.

- Non-education journals (including "popular" and "news" magazines.
- Research, including journal-published reports, dissertations, and other reports and monographs.
- Position papers, conference reports, and other documents.

(c) Meetings

Meetings which were attended included ones in Mt. Clemens, Michigan; Canton, Ohio; Washington, D.C.; Denver, Colorado; and several in Columbus, Ohio. At these meetings there was opportunity to listen to presentations, attend workshops, discuss with teachers and other educators, and review the displays of manufacturers.

(2) Collate, analyze, and synthesize the information collected.

This second stage of the project was the necessary prelude to the development of this report. Questionnaire responses were collated, literature reports were abstracted, and information from meetings and discussions was culled. Appendix A contains an annotated list of most of the literature surveyed. (It should be noted that not all of the articles and reports deal with hand-held calculators: some refer to desk calculators. It was felt that, in view of the limited amount in print on hand-held calculators plus the similarity in use of the two types of calculators, there might be some information on desk calculators which is applicable to hand-held calculators.)

Appendix B contains the complete sets of responses from questionnaires, organized by type of respondee and by question.

(3) Prepare report.

The remainder of the body of this document is the analysis of the above information. This is based on the data collected during May, June, and July 1975, which was incorporated into the Interim Report, plus an analysis of what has occurred since the Interim Report was prepared, but without further data collection. A summary is presented of the most meaningful information on:

- the availability of calculators,
- the status of the case against using calculators,
- the status of the case for using calculators,
- what is going on in schools, and
- empirical evidence.

In the summary are specific recommendations related to the use of calculators.

[Perhaps it should be reiterated that there is no statistical analysis of the responses to the information obtained from the questionnaires. The samples were not randomly selected; the sampling techniques employed were designed only to identify arguments which are being proposed for and against the use of calculators in schools. No effort was made to sample representatively to estimate the extent to which any group holds a certain position or the proportion who hold a position for a certain reason. That this type of survey should not be a goal of the project was stipulated by National Science Foundation personnel early in the development of the proposal.]

In addition to the analysis in the body of the Final Report, several position papers were prepared by persons who have devoted much thought and effort to the promises and problems posed by the use of calculators in the elementary and secondary schools of this country. The topics were identified by a small group of persons meeting in Washington, D.C. on 31 July-1 August 1975. The position papers are incorporated as Appendices C, D, E, and F:

- Appendix C: Teaching Mathematics with the Hand-Held Calculator

  by George Immerzeel, Earl Ockenga, and John Tarr

- Appendix D: Hand-Held Calculators and Potential Redesign of the School Mathematics Curriculum

  by H. O. Pollak

- Appendix E: Some Suggestions for Needed Research on the Role of the Hand-Held Electronic Calculator in Relation to School Mathematics Curricula

  by J. F. Weaver

- Appendix F: Calculators and School Arithmetic: Some Perspectives

  by Zalman Usiskin and Max Bell

II. Availability of Calculators

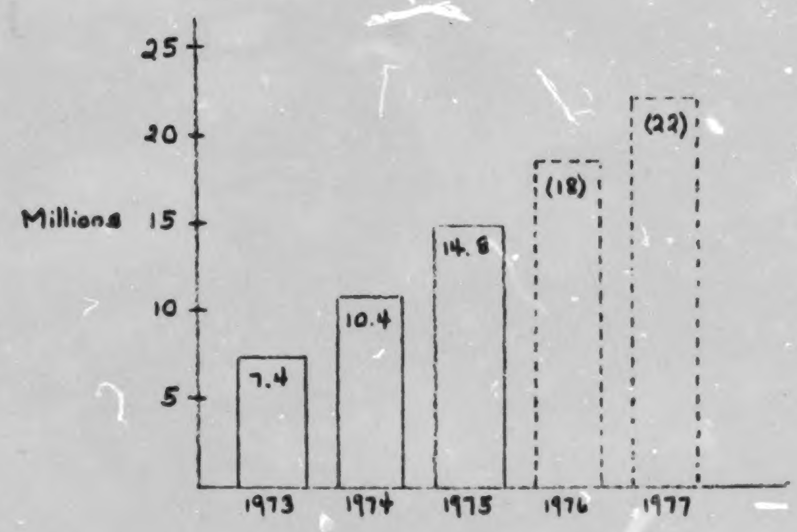
During 1975, it was evident that calculators were "selling like hotcakes". As prices tumbled early in the year, calculators lost their position as status symbols, and reached the point of being considered necessities by many. The level of sales and the implications of data projections for the schools are considered in this section.

Sales

Calculator manufacturers (with one exception) were, to say the least, reluctant to disclose information on sales. Distributors indicated inability to access this information readily. Therefore, much of the information on sales comes from published sources: marketing journals such as Discount Store News, Merchandizing Week, and Electronics regularly publish both data and projections.

For the Interim Report on this project, Shumway developed two tables to summarize such information on sales; one is on sales estimates per year:

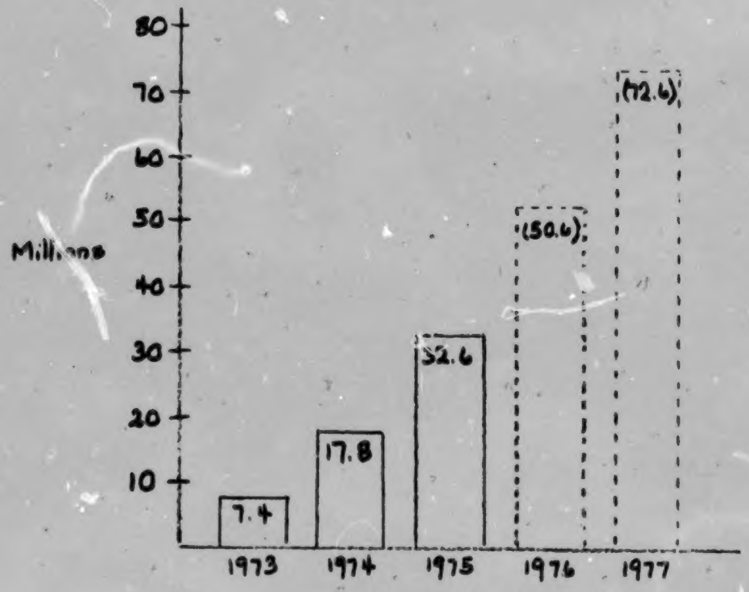
Hand-Held Calculator Sales Per Year
in the United States



That such estimated data are tenuous must be clearly understood. To illustrate this, Usiskin and Bell (1976) cite data "from a 1975 survey for industry by a reliable private research organization" (p. 2). Their data indicate sales of calculators to have been 8.34 million in 1973, 10.52 million in 1974, and 12.77 million in 1975, with a projection of 14.75 million in 1976. Another figure is provided by Electronics (8 January 1976), indicating that estimated sales are 21 million for 1976.* It is safe to say that the number of calculators being sold is large, increasing each year, and neither fully tabulated nor predictable!

The second table developed by Shumway presents the data in terms of cumulative sales estimates:

Cumulative Hand-Held Calculator Sales
in the United States



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* Shumway provided sales statistics for the top five leading magazines in 1974 (included in the 1975 Information Please Almanac), to serve as a referent for the magnitude of the figures:

TV Guide	20.1 million	Family Circle	8.3 million
Readers' Digest	18.8 million	Woman's Day	7.9 million
National Geographic	8.8 million		

The data provided by Usiskin and Bell (1976, p. 2) are slightly more conservative. Electronics (8 January 1976) indicated that "sales of handheld calculators have just about leveled in the U. S. . . . The balance of production for all types of calculators has again shifted to Japan" (p. 86). Nevertheless, they predict that 1976 should be a "solid year" for calculators.

Shumway reported that calculator sales in over half the instances are being made to housewives and students for home and school use. However, it is frequently noted in marketing reports that the actual users of calculators (especially the simpler ones) are not clearly determined; it is particularly unclear how many of the instruments are actually in the hands of students. As one visits schools and talks with teachers, it is clear that this number is increasing. Shumway also noted that more and more sophisticated models are being purchased each year (especially as cost has dropped and as familiarity increases).

Prices

There are two price ranges which seem appropriate for school use:

(1) basic four- or five-function machines with limited memory

(2) fairly sophisticated scientific calculators with functions such as x^2, $\sqrt{x}$, $1/x$, $x!$, $\sqrt[x]{y}$, y^x, log, e^x, arc, sin, cos, tan, sin h, degrees/radians, etc.

Examples of each of these include:

(1)	basic four- or five-function machines	- $10 - $20	
e.g.,	Novus 650	$ 9.95	($6.99)[*]
	Sharp EL-8005	$19.95	($13.95)
	Texas Instruments TI-1200	$19.95	($13.95)
	Rockwell 10-R		($12.66)

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[*] Prices in parentheses were observed in Columbus, Ohio in August 1975.

(2) scientific calculators	$50 - $100	
e.g., Melcor SC-635	$69.95	
Hewlett-Packard HP-21	$125.00	
Casio fx-11	$49.95	
Texas Instruments SR-50A	$99.95	($66.95)

Electronics (8 January 1976) indicated that sales of four-function, personal calculators are or will be:

1974	$265 million
1975	$268 million
1976	$276 million
1977	$301 million

In his analysis of the situation in August 1975, Shumway stated that the predicted retail sales price within one year for basic four- or five-function machines is $5. The predicted retail sales price within two years for scientific calculators is $25. He noted that most manufacturers would only make such predictions verbally. Others, however, have indicated that they believe that "rock bottom" has been reached, and that prices will either stabalize at current (February 1976) figures or begin to climb slightly for basic four-function calculators; prices for scientific calculators may, however, continue to decline (at least to some extent). This belief is based on the point that the production/marketing factors which combined to drive prices downward in 1975 parallel the situation of several years ago with television sets: prices now have nowhere to go but up, for basic four-function calculators. Refinements are still being made in the production of scientific calculators, however; for instance, one company recently placed a calculator on the market for $395, approximately half the cost of a comparable calculator from another company.

Perhaps the most pertinent point for educators is that it seems that prices of calculators appropriate for use in elementary and secondary schools will not vary so greatly that there is reason either to speed or procrastinate in purchasing calculators on economic grounds.

Current Level of Use in Schools

Estimates of the extent to which calculators are now being used in schools are highly subjective. The project survey of school personnel suggested that about two-thirds of the schools used calculators in some way: this figure, however, is biased, since many schools were identified which were known to have been using calculators. Meiring's (1975) survey of Ohio secondary schools suggested that about 18% of the schools responding had systematic use of calculators. Most of the use occurred in grades 11 and 12. Thirty-four per cent report that calculators were prohibited in some classes or for certain classroom activities (notably tests). In the past six months (since these data were collected), it would appear that more teachers and schools have become involved in using the calculator in some way: perhaps an estimate in the range of 25% to 50% is the best projection that can be made.

Estimates of Future Use

Given the data on prices, the data on the number sold, and the fact that in the project survey teacher educators, state supervisors, and textbook publishers appear to be "sold" on the desirability of the use of calculators in schools, school use of calculators is likely to increase. Factors which might tend to control the use are parent and teacher reluctance to allow calculators in the classroom. As teachers and parents use calculators themselves, their reluctance is likely to diminish. By 1977, there could be widespread use of calculators in schools; by 1979, this will almost certainly be true.

III. The Cases For and Against Using Calculators in Schools

A primary purpose of this project was to identify the positions which educators hold regarding the use of calculators in elementary and secondary schools. When the responses on the questionnaires sent to school personnel, state supervisors of mathematics, and mathematics educators are analyzed, it becomes apparent that there is much similarity in their viewpoints. And in addition, a redundancy is apparent in many published articles, for these same points are reflected over and over.

The most frequently cited reasons for using calculators in schools are:

(1) They aid in computation. They are practical, convenient, and efficient. They remove drudgery and save time on tedious calculation. They are less frustrating, especially for low achievers. They encourage speed and accuracy.

(2) They facilitate understanding and concept development.

(3) They lessen the need for memorization, especially as they reinforce basic facts and concepts with immediate feedback. They encourage estimation, approximation, and verification.

(4) They help in problem solving. Problems can be more realistic and the scope of problem solving can be enlarged.

(5) They motivate. They encourage curiosity, positive attitudes, and independence.

(6) They aid in exploring, understanding, and learning algorithmic processes.

(7) They encourage discovery, exploration, and creativity.

(8) They exist. They are here to stay in the "real world", so we cannot ignore them.

The last reason -- the pragmatic fact that they exist and that they are appearing in the hands of increasing numbers of students -- is perhaps the most compelling. *How* they can be used to facilitate each of the other seven beliefs is therefore a question that must be attacked.

The most frequently cited reasons for *not* using calculators in schools are that:

(1) *They could be used as substitutes for developing computational skills*: students may not be motivated to master basic facts and algorithms.

(2) *They are not available to all students*. Because they cannot afford a calculator, some students are at a disadvantage.

(3) *They may give a false impression of what mathematics is*. Mathematics may be equated to computation, performed without thinking. Emphasis is on the product rather than on the process; structure is deemphasized. Mental laziness and too much dependence are encouraged; lack of understanding is promoted. Some students and teachers will misuse them.

(4) *They are faddish*. There is little planning or research.

(5) *They lead to maintenance and security problems*.

The first concern -- that students will not learn basic mathematical skills -- is one expressed most frequently by parents and by other members of the lay public, as reflected (and created) by newspaper articles. But it builds a strawman, for few educators believe that children should use calculators in place of learning basic mathematical skills. Rather, there is a strong belief that calculators can help children to develop and learn more mathematical skills and ideas than is possible without the use of calculators. Much serious attention must be given by teachers and others to proving that this belief can be implemented and become fact.

For the Interim Report, Shumway considered the responses which educators gave on the questionnaires as reasons for and against using calculators, comments made in interviews and discussions with various persons, and statements in articles and other published literature. He then developed a section in which he expanded on the arguments for and against using calculators. Shumway's analysis is presented on the following pages (14-23).

Arguments for Using Calculators

The extreme of the point of view favoring the use of calculators is represented by this statement:

> "Hand-held calculators as sophisticated as the so-called 'scientific calculator' should be made readily available to all children, for all school work, from first grade on."

In support of such a position, variations of the following arguments may be cited:

(1) "There will no longer be any need for the usual paper-and-pencil algorithms for the basic operations. Algorithms are designed to carry out repetitive calculations efficiently, accurately, and without thinking. Clearly, the calculator is the best calculational algorithm available today. Paper-and-pencil algorithms might be taught for historical, cultural, or pedagogical purposes; however, few children (or adults) will choose paper-and-pencil algorithms when calculators are available."

(2) "Scientific calculators will not be expensive. The price of scientific calculators began only a few years ago at $400; currently, they are available for as little as $50. There is no reason to believe that they will not soon be available for less than $20 (which is the cost of two tanks of gas for a car). Cost will not significantly deter the widespread use of hand-held calculators."

(3) "Most children will probably learn the basic addition, subtraction, multiplication, and division facts in order to make estimations and to save time. Extensive drill and practice exercises will be unnecessary."

(4) "Decimals and scientific notation will be introduced early in first grade. Children will work with numbers such as .0285714285 and 1.893456 08. The first number is that part of a cake each of 35 children would get if it is divided into 35 equal parts; the second is the number of seconds a six-year-old child has lived. Children can and will work comfortably with such numbers. Calculators will facilitate early continuous experiences with a whole new class of numbers."

(5) "Mathematical exercises will be more realistic. Exercises will no longer have to be chosen so that there are integer solutions. So-called 'grubby numbers' and tedious calculations will be done with ease."

(6) "Calculators are fun. The motivational aspects of the hand-held calculator are exciting. Children create their own interesting problems. Low achievers generate new enthusiasm for mathematics because they finally have no fear of being unable to perform the necessary calculations. Children are eager to do mathematics when calculators are available."

(7) "The addition and multiplication algorithms for fractions can be delayed until algebra."

(8) "The calculator facilitates number sense. Because of their simplicity and speed, hand-held calculators will allow children to explore products, sums, powers, logarithms, trigonometric functions, etc., with numbers of all sizes with a frequency never before possible. Intuitive number sense will be much facilitated by such extensive, continuous, and early experience with numbers and their properties."

(9) "Hand-held calculators make calculations easy and practical for all children. It must be remembered that decimal notation, Arabic numerals, zero, paper-and-pencil algorithms, etc., were introduced not to teach mathematics, but to make calculations easier. The hand-held calculator was invented for the same reason."

(10) "Hand-held calculators stimulate interest in and facilitate the teaching of mathematical concepts. Homomorphic properties of functions, properties of logarithmic and exponential functions, characteristics of rational exponents, compounding continuous interest, combinatorics, trigonometric functions, limits, number theory, etc., can all be learned in more interesting manners because of the calculational power which the calculator provides."

(11) "The calculator can be used to facilitate problem solving. Open exploration and new problems can be offered to children because of the facilitating calculational power which the calculator provides. For example. learning to predict for what integer values of x will 1/x fill the calculator display screen teaches a great deal about our base ten numeration system and relative primes."

(12) "Hand-held calculators provide experience with the only practical algorithm which is used in society today. No business or profession carries out extensive calculations without the use of a calculator. Most family financial calculations will soon be done by calculator."

(13) "Hand-held calculators will place the emphasis on when and what operation to use rather than on how to perform the paper-and-pencil algorithm correctly."

(14) "There will be more interest in estimation. Since calculator errors tend to be dramatic rather than minute, estimating 'ballpark' answers will be useful in avoiding errors."

(15) "The power of mathematics used by the common man will increase astronomically. A simple example can be used to illustrate this. Suppose it takes $10,000 per year for a particular couple to retire today. Assuming an annual inflation rate of 5 per cent, how much per year would be required 20 years from today? The sequence (1.05, y^x, 20, x, 10,000, =) gives the answer of $26,533 in 10 seconds. Tailor-made family financial planning would be much improved by such calculational power."

(16) "More time will be available to teach mathematics in depth. Since calculators increase the speed and accuracy with which children can do calculations, much more time will be available to learn the concepts and principles of mathematics."

(17) "New topics in mathematics can be introduced into the curriculum. The calculational power of the calculator allows the consideration of new topics while the de-emphasis of paper-and-pencil algorithms produces more time for new topics."

Arguments Against Using Calculators

The extreme of the point of view against the use of calculators is represented by this statement:

> "Hand-held calculators should be banned from classroom use for mathematics."

Arguments cited in support of this position include variations of these points:

(1) "Hand-held calculators would destroy all motivation for learning the basic facts. Calculators do not remove the need to know basic facts such as 9 x 7. To raise children to run to their calculators for every simple calculation would be folly. Such dependence on calculators would be most unfortunate."

(2) "The use of calculators would destroy the basic, mainstream mathematics of the elementary-school curriculum. Society's major objective for elementary-school mathematics is that children learn the basic facts and be able to perform the paper-

and pencil algorithms for addition, subtraction, multiplication, and division. If calculators are allowed in schools, children will no longer see any need for basic calculational skills. Even banning calculators on certain days or only using them for checking would seem unfair and illogical to children. Calculators must not be used for any teaching of mathematics."

(3) "The cost of calculators prohibits their use. Schools simply cannot afford to provide calculators for children. The cost of hand-held calculators is prohibitive and their attractiveness makes them disappear all too frequently."

(4) "Calculators are particularly inappropriate for slow learners. What possible motivation would such children generate for learning an algorithm they know they can do on a calculator much more quickly and accurately? Calculators would insure that poorly motivated students would not learn the basic skills."

(5) "The child's notion of the nature of mathematics would be changed by the use of calculators. There is a real danger that if calculators are used, children will think that pushing buttons on a black box is mathematics."

(6) "The use of calculators would reduce children's ability to detect errors. We are all familiar with the belief that if a calculation was done on a calculator it must be right. Not only is such faith unjustified, but discovering errors of key-punching a calculator is almost impossible since there is no record of what was done."

(7) "Paper-and-pencil algorithms are still necessary, basic skills. Calculators can never be everywhere. Children must still be able to calculate on their own. The availability of calculators in schools would remove children's need for practicing the basic skills. Homework done at home would no longer ensure facility with the basic skills, since the home is likely to have a calculator. Schools must ban the use of calculators to ensure facility with the basic skills of arithmetic."

(8) "Batteries lose their charge and wear out. Dependency on batteries for computational arithmetic would be foolish."

(9) "The use of hand-held calculators would discourage mathematical thinking. If children can do any mathematical calculation by pressing a few buttons, problem solving will be done by guessing, not mathematical thinking. Try this, try that, keep doing things with the numbers until the answer looks right. Non-thinking guessing will become rampant if calculators are available in schools."

(10) "Parents are unalterably opposed to the use of calculators in the schools. The schools have failed miserably in the teaching of basic skills as it is. The introduction of calculators would be, in effect, not teaching mathematics at all. Schools would be exhibiting extreme political ineptness to introduce calculators."

Discussion of Arguments

It is probably obvious that the validity of the arguments in some of the statements above is questionable -- depending, to some extent, on your own viewpoint! It should also be obvious that not all proponents or opponents of using calculators in schools take extreme positions such as those identified. The following compromises have been suggested, for instance:

(1) Restrict the capability of the calculator through masking or making electronic modifications to control the operations available so that paper-and-pencil algorithms are still necessary and/or so that mathematical capabilities beyond the current curriculum are unavailable to the student.

(2) Restrict the use of calculators to checking answers only, or restrict the use to certain days of the week so that basic facts and paper-and-pencil algorithms are still necessary.

(3) Restrict the use of calculators to the upper grades (10-12), where presumably students have already learned the basic facts and the paper-and-pencil algorithms.

Some react to such compromises as an unworkable effort to "have your cake and eat it too", while others view them as examples of democratic compromise to achieve the best solution. The compromises do serve to focus attention on what appear to be the fundamental arguments regarding the hand-held calculator. The proponents' argument is essentially:

> "The hand-held calculator is the tool used in society today for calculations. Schools are 'burying their heads in sand' if calculators are not recognized and used as the calculational tool that they are."

The opponents' argument is essentially:

> "The principal objectives of mathematics instruction (at least in grades k-9) are that children learn the basic facts and the paper-and-pencil algorithms. Such learning will not occur if calculators are made available in schools."

It would seem that a rational approach to the resolution of the problem (perhaps over-simplified) would involve:

(1) Determining current and future societal needs for the basic facts and the paper-and-pencil algorithms.

   (a) If there are no needs for such skills, drop the emphasis on them and introduce the widespread use of calculators.

   (b) If there are needs for such skills, move to question 2.

(2) Can the calculator be used in the classroom and still build students' needed skills (as identified in 1b)?

Such a procedure would seem to satisfy the concerns of opponents of the use of calculators. The proponents of the use of calculators would (probably) claim that such an over-simplification of the benefits of the use of calculators is ignoring a potentially powerful educational device.

Potential Implications of the Widespread Use of Calculators

Suppose we adopt the position that there will be widespread use of the calculator in schools. What are some of the benefits and disadvantages os such widespread use?

(1) Curriculum concerns

As Pollak (1976) has aptly described in his position paper (see Appendix D), there are two partial orderings which are often used in the designing of a mathematics curriculum. For example, the mathematical development of the number systems suggests that children ought to work with addition of whole numbers before they study addition of decimals. The algorithms for addition of decimals require facility with the addition of whole numbers. Hence the order: whole numbers, then decimals, rather than decimals followed by whole numbers. Such partial orderings may be called content orderings.

A second partial ordering which must be considered in curriculum development is a social value ordering. Topics in a mathematics curriculum are included and ordered by the topic's worth to society. For example, the quadratic formula is included in the curriculum before compound interest because society views the quadratic formula as more essential for the needs of society than compound interest. Mathematically, either topic could be introduced before the other. Socially, the quadratic formula has priority over compound interest. Such partial orderings may be called societal orderings.

If the introduction of a new device such as the hand-held calculator makes a significant change in either the content ordering or the societal ordering, then major curriculum modification would seem appropriate. Such changes in the partial orderings are possible with the calculator. For example, the algorithm for addition of decimals is the same as the algorithm for addition of whole numbers: the same buttons are pushed for either. Thus, it may no longer be necessary or desirable to delay decimals until fifth grade.

A careful, extensive study of the impact of the calculator on the curriculum is needed: there appear to be significant changes which could (or ought to) be made. In their position paper, Usiskin and Bell (1976) (see Appendix F) present some initial suggestions on this task.

(2) Computational skills

The principle purpose of a calculator is to make calculations easy. Consequently all the basic operations of arithmetic, square roots, trigonometric functions, logarithms, etc., can be computed by very young children. Decisions regarding curriculum need no longer be made based on whether or not children can perform the calculations, but rather on whether they understand the concepts involved. It is probably true that the concept of square root is easier to understand than the former computational algorithms for finding square roots. It may or may not be desirable to introduce square roots much earlier in the curriculum. The point is that the decision need not be based on the difficulty of teaching paper-and-pencil algorithms for finding square roots.

Compound calculations need no longer be avoided. "How many seconds have I been living?" may be a very reasonable calculation problem for a nine-year-old (60 x 60 x 24 x 365.25 x 9). The cost per gram of various candy bars is calculationally trivial and a reasonable question to pose. There no longer need be a fear of non-integral numbers (2.7 x 5 x 17.6 is as easy as 3 x 5 x 16). Problems do not need to be artificially simplified. The numbers can be realistic.

At the minimum, one would expect some de-emphasis of the paper-and-pencil algorithms. Most calculations, in reality, will not be carried out by paper-and-pencil. It is likely that schools will begin teaching paper-and-pencil algorithms as another way to do calculations, but not the principal way.

(3) Teacher education

It is easy for the teacher educator to advocate widespread use of calculators. It is another matter for the classroom teacher actually to implement their use. The first difficulty encountered is parental opposition to calculators. The second difficulty is that the current curriculum is not designed for calculators. Exercises and problems which use the calculational power now available must currently be developed by the teacher. (Some of the articles and books cited in Appendix A contain helpful suggestions; the position paper by Immerzeel, Ockenga, and Tarr (1976) in Appendix C provides a variety of specific activities; textbook publishers plan to have some materials available by 1977.)

The third difficulty is that most teachers lack the mathematical background necessary to deal with the questions and mathematics which can be generated by the use of calculators. For example, there is a mathematically honest explanation for the sine function which can be given to first graders. Most first-grade teachers would be unable to provide such an explanation. Many junior high school teachers would be unable to provide such an explanation. And many high school teachers would have difficulty with the hyperbolic sine function.

The fourth difficulty is that techniques for teaching mathematics with calculators have not been illustrated. The effect of calculators on children's number sense and other mathematical factors is not known, either through research or tradition. Each teacher must break new ground in the interaction of calculators and children learning mathematics.

(4) Budgets

Consider a typical elementary school with two classes at each grade level. In order to provide approximately one basic four-function calculator for each two students, such a school would need to spend $1800 ($10 per calculator for 180 calculators, or 15 per class). Consider such a cost in perspective: $1800 is the cost of 30 filmstrips or 120 minutes of 16 mm film. Given the impact on students, such a cost could be defended easily. Of course, if one acquires scientific calculators, the cost would be $9000, or the equivalent of 150 filmstrips or 600 minutes of 16 mm film. It would appear that the widespread purchase of calculators may not be a major financial burden for a school which routinely purchases materials such as filmstrips or films. The data on purchases of calculators suggest that many children will soon have access to a calculator regardless of the school action. The school's responsibility will probably be to have machines available for those children who do not have access to a calculator. The cost of such a requirement could soon be relatively low.

Summary

The impact of widespread use of hand-held calculators is likely to be:

(1) A de-emphasis on paper-and-pencil algorithms.

(2) More significant and interesting mathematics in the curriculum.

(3) Consumers and decision-makers much better prepared to deal with the voluminous amount of data in communications today.

Shumway's point of view expressed above is echoed to some extent in the position papers in Appendices C through F. Each of the writers of those papers expresses additional concerns and thoughtful comments; attention is directed particularly to the papers by Pollak and by Usiskin and Bell.

IV. Ways in Which Calculators Are Now Used in Schools

Calculators are being used in various ways in classrooms scattered throughout the country. Last year, such activities were generally more extensive at the secondary-school level; this year, elementary-school teachers are increasingly introducing them for specified purposes. They are recognizing that the calculator is a part of children's lives. In many instances (perhaps too many), use of the calculator is restricted to checking the result of paper-and-pencil computation. As teachers explore potential uses, and as more specific suggestions appear in print, additional use is made of the calculator.

Two fears must be expressed:

(1) That calculators will not be used appropriately, so that few positive benefits of their use are apparent.

(2) That teachers will indiscriminately buy materials for use with calculators (as in some cases they have done with metric materials).

As Immerzeel, Ockenga, and Tarr (1976) point out, to avoid "future shock" immaginative software must be developed. They also make recommendations regarding use of calculators (p. 5):

(1) Primary level: incidental use, especially in an interest corner.

(2) Intermediate level: availability in the school of class sets for occasional use

(3) Junior high level: availability of class sets for each teacher

(4) Senior high level: a calculator for every student, available anytime

They go on to provide a variety of specific illustrations for using the calculator at each of these levels, usually within the existing curriculum.

In general, certain patterns of use are evident:

(1) The district or school purchases a small number of calculators, which are given to teachers for exploratory activities. This is followed by discussion and decision on whether the district or school should purchase more (e.g., Columbus, Ohio).

(2) Remedial mathematics or Title I classes receive calculators for use with low achievers who have not previously learned computational skills well (e.g., Washington Irving High School, New York; Berkeley, California).

(3) Calculators are placed in advanced science and mathematics classes in secondary schools (e.g., Lubbock, Texas).

(4) Exploratory work on topics for which calculators seem most appropriate is in progress (e.g., under the direction of such mathematics educators as Immerzeel, Kessner, Rudnick, Scandura, Weaver).

(5) Pilot studies and/or research is being conducted on the effect of use of calculators (e.g., with low achievers at the secondary level in Chicago; in such California schools as Cupertino, Garden Grove, Los Angeles, San Diego, San Francisco, and Santa Barbara).

Among the variety of other activities, surveys of the attitudes of teachers toward calculators and/or uses being made of calculators have been conducted (e.g., Philadelphia; Shawnee Mission, Kansas; Ohio; California); these sometimes lead to the development of policy statements. The 1975 Annual Leadership Conference at the University of Michigan focused on the role of calculators, as did several groups at a September 1975 meeting on secondary school mathematics attended by educators from throughout Ohio. Workshops were presented at local, regional, and national mathematics meetings in 1975, and plans are underway for extending such offerings during 1976 (e.g., the NCTM Name-of-Site Meeting in Detroit featured calculator workshops). The National Council of Teachers of Mathematics is developing film materials on calculators (with a grant from a calculator manufacturer). In cooperation with ERIC/SMEAC, NCTM is developing a compilation of teacher-suggested activities for use with calculators. Several journals (e.g., Instructor and Arithmetic Teacher) will have 1976 issues focused on the calculator.

Analysis of the published articles and books cited in Appendix A indicates that many fall into one of four categories:

(1) General statements about calculators: e.g., Denman, 1974; Higgins, 1974.

(2) Pros and cons: e.g., Etlinger, 1974; Fiske, 1975.

(3) Costs and features: e.g., Jesson and Kurley, 1975; Consumer Reports.

(4) Varied uses: e.g., Engel, 1974; Judd, 1975.

The annotations in Appendix A may provide a guide to locating materials of specific interest.

Usiskin and Bell (1976) (see Appendix F) take exception to merely incorporating calculator uses into the existing curriculum. For reasons which they state, "It is thus our belief that the insertion of calculators into K-6 classrooms using most existing curricula is fraught with peril" (p. 36). They argue for an alternative curriculum, and provide an assessment of how the curriculum may be restructured. They note that this may be threatening to those who view the present curriculum as optimally logical and sequential: their specific suggestions (pp. 40-49) could, however, suggest to many elementary teachers a different way of considering the use of calculators in schools.

V. Empirical Evidence

At the time that the Interim Report on this project was prepared in August 1975, attention was called to three points:

(1) At this time, there is comparatively little evidence on the effect of the use of hand-held calculators in schools. Studies have been exploratory in nature, often with the support of a calculator manufacturer.

(2) That the calculator can be used to teach certain topics seems clear: that significant achievement gains will result is not clear. As might be expected, attitudes are reported to be generally positive.

(3) Not all of the research has focused on significant questions (some has remained unpublished for this reason). There is a definite need to establish priorities and attack the questions that can and should be answered by research.

As this Final Report is prepared in February 1976, the same three points can be reiterated. The research picture has not essentially changed.

As Weaver (1976) has pointed out in his paper on needed research (see Appendix E):

> . . . The very newness of calculators provides little of a research base upon which to build. . . . The extent of ongoing research is very difficult to assess; this also is true of the nature of that research. We are given hints from the brief progress reports released by some projects . . . but by and large we have precious little information--and none of it definitive--regarding the extent and nature of ongoing research. (p. 18)

He goes on to express the thought that "When the annotated listing [of research on mathematics education] for calendar year 1975 is compiled, more calculator investigations are bound to appear; but there still will be no plethora of such investigations reported" (p. 18). Very true:

there has been no plethora; research evidence remains scarce (perhaps the 1976 listing will contain more?).

There is so little research on the use of hand-held calculators, in fact, that there is some question if it is worthwhile to review it. Most of the studies barely meet criteria for being termed "research -- "action research", "preliminary study", "inquiry", "exploration" are the words which investigators use in the published reports. There are rumors of many studies going on: most of them turn out to be explorations to find out what can be done with calculators. Some of the "hardest" data come from studies conducted by calculator manufacturers; not surprisingly, these indicate that students (a) can use the calculator with a variety of content and (b) achieve well when using the calculator. Many schools are checking data on their own students to find out the effect of the use of calculators with their students: this is a highly appropriate activity -- providing it continues as new suggestions and materials for using the calculator appear. Schools must be wary of selecting options too quickly -- of deciding that this way of using calculators is effective and that way isn't -- before the range of options (that is, a diversity of material designed for calculator use) has been developed.

Over the years, several dozen studies have been conducted with desk calculators. There has been some thought that this research might provide some useful information which would be applicable to hand-held calculators. Alas, the studies are not all designed as well as they should have been. On Table 1, 21 of the studies with desk calculators are summarized; note should be made of the small sizes of samples, the short lengths of time, the limited purpose of many of the studies, the evident confounding of variables.

TABLE 1

REPORTS OF RESEARCH ON DESK CALCULATORS

Author	Date	Grade level	N	Length	Type of research	Calculator use	Findings
Advani	1972		18 pupils (EMRs, NIs)	6 months	action	checking	significant increase in achievement, positive attitudes
Beck	1960	4-6	"several classes"		action (no data)		students can use calculators, motivation improved
Betts	1937	6	13 pupils	8 weeks	exploratory	doing examples	increased achievement and interest
Broussard *et al.*	1969	7-9	(low achievers)				significant achievement gain, more tackled more mathematics
Buchman	1969	sec.	1185 schools	(N.Y.)	survey	-	13% of schools had calculators
Cech	1970, 1972	9	4 classes (general mathematics)	7 weeks	experimental	checking	no gains in skills or attitudes (therefore must be used earlier, longer)
Durrance	1965	6-8	70 pupils	9 weeks			calculator-use had no effect except on reasoning in grade 7
Ellis and Corum	1969		2 classes (low achievers)		experimental		no significant difference in achievement; better attitudes
Fehr *et al.*	1956	5	8 classes	4-1/2 mos.	experimental	doing multiplication	calculator group was better on tests of computation and reasoning, but not overall mathematics achievement; attitudes were positive

TABLE 1 (Continued)

Author	Date	Grade level	N	Length	Type of research	Calculator use	Findings
Findley	1967	9		1 year		working (with 2 types of textbook)	calculator plus traditional textbook resulted in better achievement only on arithmetic fundamentals when compared with calculator plus calculator-specific textbook
Gaslin	1972, 1975	9	6 classes (general mathematics)		experimental	type of algorithm for operations with rational numbers	no significant differences in achievement, transfer, retention, attitude, or rate (therefore calculator algorithm is viable alternative to conventional algorithm with or without calculator)
Hohlfeld	1974	5	79 pupils			immediate feedback on multiplication	daily drill with calculators resulted in higher achievement than drill with paper/pencil or "regular activities"
Johnson	1971	7		1 unit	experimental	working, checking	differences between calculator and non-calculator groups found on both achievement and attitude measures
Keough and Burke	1969						calculator group had better achievement than control group
Ladd	1974	9	201 pupils (10 classes) (low achievers)		experimental	working,	no significant differences between calculator and control groups was found

TABLE 1 (Continued)

Author	Date	Grade level	N	Length	Type of research	Calculator use	Findings
Longstaff *et al.*	1968	5,9	2 groups				attitudes were positive
Mastbaum	1969	7-8	8 classes (slow learners)	6 months	experimental	working	calculator-use did not significantly improve attitude, or increase mathematics ability, non-calculator computational skill, mastery of concepts, or problem solving ability -- but no significant differences were found between groups
Schott	1955	4-9		4 months	experimental	solving problems	calculator group scored higher than control group; problem-solving ability increased
Shea	1974	4		30 weeks	experimental	flow charting	calculator group scored higher than non-calculator group on computation but not other achievement tests or attitude
Stocks	1972		15 pupils (EMRs)	15 lessons		working	EMRs capable of doing division with calculator; both groups improved
Triggs	1966	primary	1 class	9 weeks	action		both groups improved

There appear to be very few potentially transferable findings from the studies with desk calculators:

(1) Children can learn to use calculators.

(2) Children generally enjoy using calculators.

(3) Low achievers may profit from using calculators, but calculator use should not be restricted to low achievers.

(4) Calculators can be used for checking paper-and-pencil computation.

(5) Calculators may or may not facilitate particular types of achievement.

On Table 2, 8 studies on hand-held calculators are summarized. A few additional comments about each might aid in making readers further aware of the limitations of the research information on hand-held calculators.

Comments on Research Reports on Hand-held Calculators

The major goal of the study reported by Hawthorne and Sullivan (1975) was to "discover how (and if) the calculators could enrich, supplement, support, and motivate the regular program. There was no intent to change the program to fit the calculator" (p. 29). Barrett and Keefe (1974) expanded on some of the ways the students used the calculator: to check answers and in working with verbal problems, means, probability, palindromes, functions, and multiplication with decimals.

A comparison with a matched group indicated that the mean scores of students using calculators were higher ($p < .02$) on the concepts and computation sections of the test than were the corresponding scores for students not using calculators, but the two groups performed about equally well on the problem-solving section of the test. Two comments of interest

TABLE 2

REPORTS OF STUDIES WITH HAND-HELD CALCULATORS

Author	Date	Grade level	N	Length	Type of research	Calcultor use	Findings
Bitter and Nelson	1975	4-7			action	working, checking, games	calculator group achieved better than non-calculator group, had better attitudes
Hawthorne and Sullivan (Barrett and Keefe, 1974)	1975	6	96 pupils	1 year	action	working, checking	calculator groups higher on concepts and computation, not as high on problem solving; interest sustained
Kelley and Lansing	1975	7-8	8 classes	1 year	experimental	remedial	calculator group achieved higher than skill group on reasoning, computation
Kessner	1975	k,1			field test	working	children learn using calculators
Meiring	1975	7-12	111 schools (Ohio)	-	survey	-	calculators systematically used by 18.9%; 35.1% had guidelines; 34.2% prohibited calculator use
Schafer, Bell, and Crown	1975	5	5 classes (120 pupils)	2 days	preliminary study		calculator group scored significantly higher on calculator examples; no differences on non-calculator examples
Spencer	1975	5,6	84 pupils	8 weeks	experimental	doing computation worksheets	calculator group better than non-calculator group on reasoning in grade 5, on computation and total test in grade 6
Weaver	1976	2,3,5	7 classes	3 years	exploratory	operations, number sentences	pupils encounter no consequential problem using calculators

are made by Hawthorn and Sullivan:

> Project evaluators do not believe that calculators have any great inherent ability to support and motivate mathematical study, though these instruments definitely have some powerful computational capability. (p. 31) [Oh?]
>
> Perhaps another study can shed some light on what effect calculators have on learning mathematics if used by children without any particular direction by teachers. (p. 31) [Why?]

One wishes that there were some discussion of each of these points in the article.

Bitter and Nelson (1975) developed a "diagnostic remediation mathematics curriculum utilizing the hand-held calculator". A group using this curriculum was compared with a group using a commercial hand-held calculator mathematics remediation program and with two groups using the "normal" curriculum, one with calculators available and one without calculators. While no data were reported in the article, the authors note that " . . . all three calculator approaches provided for significant statistical gains in both the cognitive and attitudinal domain as opposed to the control group."

Analysis of data from Project Equip, a mini-calculator program for teaching mathematics sponsored by Berkeley schools, was reported by Kelley and Lansing (1975). Two seventh-grade and two eighth-grade mathematics classes for low achievers were involved. Neither experimental nor control classes showed statistically significant gains on the CTBS (the experimental group mean was 4.87 on the October pretest, 4.98 on the May posttest; the control group's respective scores were 5.29 and 5.30). On the CTBS computation subtest, the calculator group did significantly better, however (4.9 for the control group, 6.5 for the experimental group). And on the NLSMA Reasoning Test, the gain of the control group was significant at the .08 level (0.9 points); the calculator group gained 1.9 points, which was significant at the .001 level.

Kessner (1975) presented information on a primary mathematics project designed "to research, develop, and field test activities in which kindergarten and first-grade children can use hand-held electronic calculators to promote their mathematics learning". Simplified calculators are "coupled with gamelike modules" to teach the "complex aspects of counting and the operations of addition and subtraction". In the project information cited, no data are reported.

Schafer, Bell, and Crown (1975) report on a limited "inquiry" in which one group of fifth-grade pupils was given a pretest, and then given calculators for two (2) days, with problems to work and encouragement to ask questions about the calculator. A week later a posttest was given. Overall there were no significant differences. Those who had calculators, however, scored significantly higher on examples on which the calculator could be used, while there were no significant differences on non-calculator examples (although the score of the calculator group was lower than that of the non-calculator group). One wonders whether the same statistical result could be replicated: the conditions were obviously loose. The comments in Appendix A of the Usiskin and Bell position paper (1976) report further explorations with about 20 teachers. The work does not appear to be systematic, however: it is purely exploration to find out how pupils (and teachers) react.

Weaver (1976), on the other hand, has since 1973 been systematically exploring the use of various calculators at several grade levels, in ways which connect with his (and his students') previous research on mathematical sentences and properties of operations. Although he reports some empirical data, "the principal intent of the project to date has not been hypothesis formulation and testing," but informal exploration as a necessary stage to precede controlled experimentation. This "exploratory work was independent of ongoing mathematics programs . . .", although this year "teachers are opting to have pupils use the calculator from time to time in connection with the ongoing mathematics programs". Among the emphases of the exploratory work have been chaining, doing and undoing, and related number sentences.

Spencer (1975) investigated the effect of using calculators on computation skill, reasoning ability, and total arithmetic achievement, as measured by the Iowa Test of Basic Skills. Forty pupils in grade 5 and 44 pupils in grade 6 were randomly assigned either to a group using calculators or to a group using paper and pencil without calculators. For eight weeks, both groups worked with computation worksheets prepared by the experimenter; unfortunately, the abstract of the study does not indicate the nature of these worksheets. At the fifth-grade level, the only significant difference found was on the reasoning test, favoring the calculator group. In grade 6, significant differences favored the calculator group on the computation test and on the total test; a tendency for the calculator group to have higher scores on the reasoning test was also noted.

The locations at which this research and development work is being conducted are diverse: Arizona, California, Illinois, Iowa, New York, Wisconsin. In other states, other projects are going on, with no published results as yet. For instance, Rudnick (Eye on Education, 1975) is currently directing a project in Pennsylvania with seventh graders to investigate the effects of the availability and use of the calculator on achievement and attitudes. Capoferi and Winowski (1975) present a design for a study to be conducted in Michigan schools. One hopes that carefully designed research will be planned at many other locations.

Needed Research

In his position paper in Appendix E, Weaver discusses some of the research questions which should have some priority. He called attention to the point that

> the greatest thing we have to fear today about the calculator vis-á-vis school mathematics curricula is the degree of fear that already exists about the calculator vis-á-vis school mathematics curricula. (p. 2)
>
> To many persons the calculator threatens to violate certain tenets regarding school mathematics learning and instruction--tenets that are adhered to more tenaciously than I might have expected. Suggestions for calculator uses are made within the constraints of those tenets . . . and any research that might be implicit in such suggestions would be similarly constrained.
>
> Some other persons, however, appear to be willing--possibly even anxious--to suggest calculator uses that may challenge certain of our cherished tenets. (p. 5)

Weaver distinguishes between three types of curricula -- calculator-assisted, calculator-modulated, and calculator-based -- and points out that "research should not be unmindful of such differential roles". After citing the research questions included in the NACOME Report (1975), he discusses six others for which answers should be sought. Each in turn can lead to a series of investigations.

Summary

Evidence on the effectiveness of calculator use is largely experiential. A concise summary of the suggestions for research which should be conducted to determine the potential and problems of calculator use in schools includes investigations related to:

- when and how to introduce calculators
- effective procedures for learning basic facts, computational skills, problem solving, and various mathematical ideas

- effective calculator algorithms
- long-range effects of using calculator algorithms
- need for paper-and-pencil algorithms
- effect of calculator use with specific content and curricula
- effect of curricula sequence/emphasis changes
- relationship between work with calculators and computers
- changes in teacher education curricula
- optimal calculator designs

One very specific caution must be emphasized: attempts at restructuring the curriculum, either extensively or minimally, must not proceed independently of research. The two are integrally interwoven, and one cannot be effective without the other.

VI. Some Recommendations

A variety of recommendations has been incorporated at many points in this Report. Many others are specified or suggested in each of the position papers. These are generally stated at the end of this section.

But first, some recommendations which have not been cited previously will be listed. These were given by educators in response to a question on several questionnaires in the survey conducted early in this project. They range from the general to those specific to the curriculum.

Recommendations from Educators Surveyed

1. Experiment and plan.
    a. Learn to use calculators yourself first, finding meaningful ways to use them.
    b. Use calculators with students only after considerable thought as to how, when, and why.
    c. Develop a school-wide policy and guidelines.
    d. Develop ways to incorporate calculators into the existing curriculum, and develop new curriculum as necessary.
    e. Plan a reasonable inservice program, evaluation, and research.
    f. Use in early grades with care, if at all.
2. Survey available calculator models carefully and buy good equipment, commensurate with student needs. Make sure that all students have access to a calculator.
3. Change teaching emphases to concept development, algorithmic processes, when to apply various operations, and problem solving using real-life and interdisciplinary applications.

4. Do not ignore the development of computational skill.
5. Think of calculators as a tool to extend mathematical understanding and learning by making traditional work easier. The focus can be on process because the product is assured.
6. Place more emphasis on problem-solving strategies. Use practical, realistic, significant problems, and more applications.
7. Spend less time on computational drill, more time on concepts and the meaning of operations. Use more laboratory activities where computation is involved but the emphasis is on learning mathematical concepts. Decrease the use of tedious, complicated algorithms; emphasize algorithmic learning, including student development of algorithms.
8. De-emphasize fractions, and emphasize decimals, introducing them earlier.
9. Emphasize estimation and approximation (including mental computation skills), checking and feedback, exploration and discovery.
10. Do more and/or earlier work with such ideas as place value, the decimal system, number theory, number patterns, sequences, limits, functions, iteration, statistics, probability, flow charting, computer literacy, large numbers, negative numbers, scientific notation, data generation, and formula testing.

Two points should be made in connection with the above recommendations:

(1) There was not consensus on all of them, nor were they all cited with equal frequency. A selection process occurred, which may reflect the beliefs of the author of this Report.

(2) The overlap of the recommendations with statements in other published materials is evident.

Major Recommendations

1. A thorough analysis of the mathematics and other appropriate curricula of elementary and secondary schools should be conducted to determine:
    - a. how calculator use could be optimally integrated with existing curricula (see Section IV, Immerzeel et al.).
    - b. how curricula should be revised/redeveloped to incorporate optimal use of calculators (see Section III, Pollak, Usiskin and Bell).
2. A careful plan for systematic research should be developed (see Section V and Weaver).
3. Following the above steps, appropriate research related to, and development of, curricula should be initiated.
4. Experiences for teachers at both inservice and preservice levels should be provided, to aid them in using calculators with students.
5. Information about research and development efforts must be communicated (with speed and accuracy) to parents and other non-educators, as well as to educators.

These recommendations are based on the assumption, derived from analysis of information secured during the project, that calculators are increasingly being accepted as an instructional tool (by both teachers and parents). Therefore an immediate need exists for sound and substantial research and development efforts.

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Fiske, Edward B. Educators Feel that Calculators Have Both Pluses and Minuses. *The New York Times*, Section IV, p. 7; January 5, 1975.

Gaslin, William Lee. A Comparison of Achievement and Attitudes of Students Using Conventional or Calculator-Based Algorithms for Operations on Positive Rational Numbers in Ninth-Grade General Mathematics. *Dissertation Abstracts International* 33A: 2217; November 1972.

Gaslin, William L. A Comparison of Achievement and Attitudes of Students Using Conventional or Calculator-Based Algorithms for Operations on Positive Rational Numbers in Ninth-Grade General Mathematics. *Journal for Research in Mathematics Education* 6: 95-108; March 1975.

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Keough, John J. and Burke, Gerald W. *Utilizing an Electronic Calculator to Facilitate Instruction in Mathematics in the 11th and 12th Grades*. Final Report. July 1969. ERIC: ED 037 345.

Kessner, Arthur. Exploring Calculators in Primary Mathematics. Berkeley, California: Regents of the University of California, 1975.

Ladd, Norman Elmer. The Effects of Electronic Calculators on Attitude and Achievement of Ninth Grade Low Achievers in Mathematics. (Southern Illinois University, 1973.) *Dissertation Abstracts International* 34A: 5589; March 1974.

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Mastbaum, Sol. A Study of the Relative Effectiveness of Electric Calculators or Computational Skills Kits in the Teaching of Mathematics. (University of Minnesota, 1969.) *Dissertation Abstracts International* 30A: 2422-2423; December 1969.

Schafer, Pauline; Bell, Max S.; and Crown, Warren D. Calculators in Some Fifth-Grade Classrooms: A Preliminary Look. *Elementary School Journal* 76: 27-31; October 1975.

Schott, A. F. Adventure in Arithmetic. *Educational Screen* 34: 65-67; 1955.

Shea, James Francis. The Effects on Achievement and Attitude Among Fourth Grade Students Using Calculator Flow-Charting Instruction vs. Conventional Instruction in Arithmetic. (New York University, 1973.) *Dissertation Abstracts International* 34A: 7499; June 1974.

Spencer, JoAnn Nora. Using the Hand-held Calculator in Intermediate Grade Arithmetic Instruction. (Lehigh University, 1974.) *Dissertation Abstracts International* 35A: 7048-7049; May 1975.

Stocks, Sister Tina Marie. The Development of an Instructional System Which Incorporates the Use of an Electric Desk Calculator as an Aid to Teaching the Concept of Long Division to Educable Mentally Retarded Adolescents. (Columbia University, 1972.) *Dissertation Abstracts International* 33A: 1049-1050; September 1972.

Triggs, E. The Value of a Desk Calculating Machine in Primary School Mathematics. *Educational Research* 9: 71-73; November 1966.

Weaver, J. F. *Calculator-influenced Explorations in School Mathematics: Number Sentences and Sentential Transformations I, II*. Project Paper 76-1. Madison: Wisconsin Research and Development Center for Cognitive Learning, January 1976.

Calculators. Consumer Reports 40: 533-541; September 1975.

The Mini-Calculator Project. Eye on Education 4: 8-9; Number 1, 1975. (Temple University College of Education.)

Overview and Analysis of School Mathematics Grades K-12. Washington: Conference Board of the Mathematical Sciences, National Advisory Committee on Mathematical Education (NACOME), November 1975.

World Electronics Markets: Mounting a Push Toward a Slow Modest Recovery. Electronics 49: 83-106; January 8, 1976.

Appendix A

Annotated List of References on Calculators in Pre-College Education

Advani, Kan. *The Effect of the Use of Desk Calculators on Achievement and Attitude of Children with Learning and Behaviour Problems. A Research Report*. December 1972. ERIC: ED 077 160. 10 pages.

Eighteen students (ages 12 to 15) used four calculators for six months to check mathematics problems. Comparisons of pre- and posttest data indicated significant increases in student interest and positive attitudes toward mathematics, while disruptive behaviors decreased.

Albrecht, Robert L. and others. The Role of Electronic Computers and Calculators in Mathematics Instruction. In *Instructional Aids in Mathematics* (edited by Emil J. Berger). Thirty-fourth Yearbook of the National Council of Teachers of Mathematics. Washington: The Council, 1973. Pp. 181-187.

Various types of computing devices are described, and their usefulness in the classroom discussed. Scant attention is devoted to calculators in general, and none to the hand-held calculator as a distinct instructional aid. The authors state that electronic calculators are "far more powerful problem solving tools than conventional machines."

Asmus, Paul. Calculators vs. Minis. *Datamation* 18: 55-58; April 1972.

A summary of key features to compare when deciding on either a programmable calculator or a minicomputer are listed. Consideration is given to the kinds of work to be done, flexibility needed, experience of users, operating features, and price.

Bahe, L. W. Finding Logarithms and Antilogarithms with a Simple Calculator. *School Science and Mathematics* 74: 221-224; March 1974.

The availability of simple, low-cost electronic calculators has removed some of the drudgery from calculations needed in the chemistry laboratory. Since logarithmic calculation is important for many experiments, a method of finding these values using a simple matrix and a four-function calculator is provided.

Barnes, Bart. Hand Calculators Cause Math Teachers' Debate. *Washington Post*, pp. 1, 8; December 16, 1974.

Pros and cons on the use of the calculator in the classroom are discussed.

Barrett, Ray and Keefe, Michael. *Using Hand-Held Calculators in Sixth Grade Classes*. Albany, New York: The State Education Department, Bureau of Mathematics Education, 1974.

Each student in the authors' sixth-grade classes was assigned a calculator for use throughout the year. Provisions for maintenance, security, and classroom organization were made. The calculators permitted exploration of a variety of topics; word problems, arithmetic means, probability, palindromes, functions, and decimal multiplication were particularly successful. The calculators motivated and strongly supported instruction on some topics. Student interest was sustained throughout the year; they accepted the calculator as a tool to help them become more independent and proficient in mathematics.

Beakly, George C. and Leach, H. W. The Slide Rule, Electronic Hand Calculator, and Metrification in Problem Solving (Third edition). New York: Macmillan, 1975.

A comparison of reverse-Polish and algebraic logic is given, in addition to descriptions of the three major types of hand-held calculators currently on the market.

Beck, Lois L. A Report on the Use of Calculators. Arithmetic Teacher 7: 103; February 1960.

Fourth-, fifth-, and sixth-grade classes used Monroe Educator calculators; these calculators perform the basic operations in much the same way they are done with paper and pencil. Although complete results were unavailable at the time of writing, several observations were made. The calculators could be operated by the students; when used as a regular classroom tool, they tended to motivate and reinforce understanding and achievement in basic skills. Children seemed to enjoy using the calculators, and to exhibit better work habits. Place-value concepts were reinforced.

Berg, Gary. Teaching Flow-charting Through Electronic Calculators. Business Education Forum 24: 30; January 1970.

Flow-charting symbols and a flow-chart describing a use of the calculator are presented, in relation to the use of calculators in businesses.

Berger, I. Calculators Get Smaller, Smarter, and Cheaper. Popular Mechanics 142: 70-75; December 1974.

Features to look for and what models have those features are included in this guide to buying a hand-held calculator.

Berger, I. Electronic Calculators: How to Choose the Right One. Popular Mechanics 139: 86-90, 192; February 1973.

What features to look for when purchasing an electronic calculator, how much to spend, and consideration of one's particular needs are discussed. Both desk and hand-held calculators are considered.

Berkowitz, S. A Rational Number Calculator. Datamation 16: 106; 1973.

A description is given of how a calculator might be built to save and display a quotient of two rational numbers regardless of length.

Betts, Emmett A. A Preliminary Investigation of the Value of a Calculating Machine for Arithmetic Instruction. *Education* 58: 229-235; December 1937.

The effect of practice with a calculating machine on the pupil's problem-solving techniques and computational skills was studied. Thirteen pupils in the second half of sixth grade completed the year's work in the six-week treatment period. Gain scores from four tests were compared, with improvement found in each case. Pupils were able to analyze more problems in the time available than they usually did.

Birtwistle, Claude. Some Further Comments on Electronic Calculators. *Mathematics Teaching* 66: 27-30; March 1974.

The author comments that pocket calculators are too easily lost, and their keys and displays are too small; small portable desk calculators are recommended instead. Several models are available for performing the four basic operations; the order in which keys are punched to perform calculations varies with the different machines, and the unnaturalness of some orders can cause problems for some children. For low achievers, electronic calculators are probably not as effective as the old type. For average and above-average students, however, the calculator presents advantages in speed and accuracy. By taking the tedium out of complex computations, it can encourage thought about the answer. Calculator use allows complex topics to be introduced at lower levels in the curriculum.

Bitter, Gary G. and Nelson, Dennis. Arizona Migrant Education Hand-Held Calculator Project. *Migrant Educator* 1: 1-3; 1975.

Evidence is presented that hand-held calculators can be used successfully in remedial mathematics programs.

Bolder, Jacqueline. Calculator Use Doesn't Add Up, Maryland NAACP Says. *Washington Star*, October 26, 1975.

The Maryland state chapter of the NAACP urged the restriction of the use of hand calculators and computers in most state classrooms.

Bormann, Thomas M. Electronic Calculating: Processing of Thoughts. *Journal of Business Education* 48: 302-303; April 1973.

Some characteristics of electronic calculators which make them desirable tools for office workers are speed and accuracy, greater capability and decreasing price, quietness and dependability, portability, and relative ease with which their use is learned. The basic objectives of calculator instruction in the business curriculum are rapid and accurate operation, and integration of a working knowledge of the calculator into the problem-solving process. Several mathematical and business concepts must be understood in order to achieve these objectives. Guidelines for preparing electronic calculator instruction calls for emphasis on both intellectual and motor skills, and simulation of actual business applications.

Braun, Alexander E. Eulogy for a Slide Rule. Science Digest 79: 65-67; February 1976.

The "passing" of slide rules is humorously decried.

Broussard, Vernon; Fields, Albert; and Reusswig, James. A Comprehensive Mathematics Program. AV Instruction 14: 43-44, 46; February 1969.

A program for low achievers in grades 7-9 from disadvantaged areas which emphasized real-world applications and use of flow charts, calculators, and other materials, resulted in significant achievement gain. Sixty per cent of the students who had participated in the program continued to take mathematics courses, compared with 40 per cent in a control group.

Buchman, Aaron L. The Use of Calculators and Computers in Mathematics Instruction in New York State High Schools. School Science and Mathematics 69: 385-392; May 1969.

Only 13 per cent of the schools reported (in 1967-68) having calculators in the mathematics department, with 2 per cent of these having computer features. Five per cent of the schools had computer facilities which were used by mathematics classes.

Buchwalter, L. Now It's Pocket Calculators. Mechanics Illustrated 69: 108-109; February 1973.

The role of calculators is explored, with comments on their potential uses.

Buckwalter, L. 100 Ways to Use Your Pocket Calculator. Greenwich, Connecticut: Fawcett Publications, 1975.

Suggestions for particular ways to use the calculator.

Budlong, Thomas S. What to Look for in a Microcalculator. Machine Design 44: 155-161; November 14, 1972.

A checklist of features to consider when choosing a hand-held scientific calculator are listed. A description of their features is given, including a summary of the characteristics of 17 representative engineering calculators.

Cantor, Charles B. Now That the Electronic Calculator Fits in Your Pocket, How Will It Fit in Your Math Class? *Business Education World* 55: 29; November/December 1974.

Future uses of the calculator in business classrooms are discussed, with comments on such topics as programming, artificial intelligence, and electronic classrooms.

Capoferi, Alfred and Winowski, Eugene. Macomb Intermediate School District, Van Dyke Schools Cooperative Project - Exploration of Classroom Use of the Hand Calculator in Grades 4-6. Mt. Clemens, Michigan: Macomb Intermediate School District, 1975.

The project is briefly described, and many examples of mathematics topics appropriate for the calculator are provided.

Cech, Joseph Philip. The Effect the Use of Desk Calculators Has on Attitude and Achievement in Ninth-Grade General Mathematics Classes. (Indiana University, 1970.) *Dissertation Abstracts International* 31A: 2784; December 1970. (See also ERIC: ED 041 757.)

Cech, Joseph P. The Effect of the Use of Desk Calculators on Attitude and Achievement with Low-Achieving Ninth Graders. *Mathematics Teacher* 65: 183-186; February 1972.

The two main reasons for using calculators with low achievers in mathematics classes are motivation and achievement. This study of calculator effectiveness involved two teachers each teaching a calculator section and a regular section of general mathematics for seven weeks. Students in the experimental group were encouraged, but not forced, to check answers with the calculators. All classes were given pre- and posttests of attitude and achievement. Results did not support the hypothesis that students using calculators would show positive gains in attitude toward mathematics, or increased paper-and-pencil computational skill. Students could compute better with the calculator than without it, however.

Clarke, John. Educational Technology. *Mathematics in School* 1: 30-31; November 1971.

Principles on which the use of educational technology is based are discussed.

Comarow, Avery. Practical New Uses for Pocket-Money Calculators. *Money* 13: 88-91; October 1974.

Different models of pocket calculators are compared, and some ways to use the calculator to solve business-oriented problems such as compound interest and taxes are presented.

Comarow, Avery. Maximizing Your Minicalculator. Money 14: 52-53; January 1975.

The total number of pocket calculators sold in the U. S. in 1974 probably exceeded 13 million. To be reasonably flexible for household uses, calculators should have separate keys for +, -, x, ÷, and =, a floating decimal point, and a display of at least eight digits. Methods for basic calculations, computation of interest, and anticipating monthly payments are presented in step-by-step formats.

D'Aulaire, Emily and D'Aulaire, Ola. Put a Computer in Your Pocket. Reader's Digest 107: 115-118; September 1975.

Reasons for buying a calculator are included in this article.

Denman, Theresa. Calculators in Class. Instructor 83: 56-57; February 1974.

The author comments on the fact that calculators are fast becoming accepted as necessary household appliances. When the children in school today are adults, they will look on calculators as being as necessary to everyday life as telephones.

Dohleman, L. What to Look For in an Electronic Calculator. Business Education Forum 27: 32-33; March 1973.

Features to consider when buying calculators for business or business education purposes include output type, decimal control, automatic rounding, portability, and programmability. A buyer of calculators for educational use should know the terminology pertaining to the machine, as well as his educational objectives. He should test the machine with the types of problems he will be using in class.

Douglas, John H. Computers 1: From Number Crunchers to Pocket Genies. Science News 108: 154-157; September 6, 1975. (Parts 2 and 3 appear in the September 13 and October 4 issues.)

Computer developments are discussed, with some mention of hand-held calculators.

Durrance, Victor Rodney. The Effect of the Rotary Calculator on Arithmetic Achievement in Grades Six, Seven, and Eight. (George Peabody College for Teachers, 1964.) Dissertation Abstracts 25: 6307; May 1965.

From grades 6-8 in a single school, 35 pairs of students were matched according to IQ and grade placement in arithmetic. One from each pair was then selected to use the calculator. Analysis of data from the nine-week study indicated that in computation, reasoning, and concepts, the calculator had no effect except in the area of reasoning in grade 7.

Edelman, Ron and Oleson, Clif. *Creative Calculatoring*. Eugene, Oregon: Action Math Associates, 1975.

Activities with the calculator are presented.

Ellis, June and Corum, Al. *Functions of the Calculator in the Mathematics Laboratory for Low Achievers*. 1969. ERIC: ED 040 847. 46 pages.

An experimental and a control class were administered pre- and posttests to check the effects of calculator use on the achievement, attitude, and academic motivation of low achievers. The use of printing calculators did not produce a statistically significant change in mathematics achievement. More favorable attitudes and weaker academic motivation were recorded for both groups at the end of the experiment.

Engel, C. William. Meet Minnie Calculator! In *Fostering Creativity Through Mathematics* (edited by Betty K. Lichtenberg and Andria P. Troutman). Tampa: Florida Council of Teachers of Mathematics, 1974. Pp. 69-77.

Appropriate topics and activities are suggested for using the calculator at a variety of levels.

Etlinger, Leonard. The Electronic Calculator: A New Trend in School Mathematics. *Educational Technology* 14: 43-45; December 1974.

The existence of low-cost calculators is not sufficient justification for their use in mathematics programs. Proponents of calculator use in schools generally have one of two contrasting views. The functional position advocates the use of the calculator to perform computations, obviating the need to teach computation. The basic tenet of the pedagogical position is that the calculator can facilitate (rather than replace) learning. The implications of this point of view, and the potential pedagogical value of the calculator are not yet known, and many specific questions need answers. Examples of the pedagogical use of calculators are described.

Feder, Chris Welles. Children and Calculators: Do They Add Up? *McCall's* 102: 34; May 1975.

Some current uses of calculators in schools in East Greenbush, New York, Virginia, and Brooklyn are described.

Fehr, Howard F.; McMeen, George; and Sobel, Max. Using Hand-Operated Computing Machines in Learning Arithmetic. *Arithmetic Teacher* 3: 145-150; October 1956.

A controlled experiment on learning multiplication by using a two-digit multiplier was conducted for a two-week period. No significant difference was found in the performance of students in experimental and control groups. However, the experimenters felt that longer use of the devices might have produced an effect, and therefore conducted a half-year experiment using the Monroe Educator model hand-operated calculator. Students using this machine made significant gains in both computation and reasoning. Although their gains were greater than those of a control group, these differences were not statistically significant. Both students and teachers using calculators had a very positive attitude toward calculator use in the mathematics classroom.

Feldzamen, A. N. and Henle, Faye. The Calculator Handbook. New York: Berkley Publishing Co., 1973.

This book gives the layman an introduction to the pocket calculator. It includes tips on how to use the calculator to solve everyday problems such as use in shopping and computing taxes.

Field, R. Science and the Consumer: Figuring Pocket Calculators. Science Digest 77: 85-86; March 1975.

Ways in which the calculator can aid the consumer are discussed.

Fielker, David. Electronic Calculators: A Changing Situation. Mathematics Teaching 65: 28-32; September 1973.

Electronic calculators are here to stay, and far-sighted schools are beginning to be interested in them. As they become more popular, it will be difficult to justify teaching as much rote arithmetic as is common. Computing devices of the past have often been effective because they involve tactile and visual experience of mathematical operations; electronic calculators do not afford one this experience. Use of the hand calculator in schools will require emphasis on place value, magnitude, and notation. Some advantages of the machines are the ability to handle large numbers, their usefulness for activities such as estimating square roots, and the possibility of using them in the design of new algorithms. Some advice on the purchase of calculators and a glossary of calculator-related terms is included in this article.

Findley, Robert Earl. An Evaluation of the Effectiveness of a Textbook, Advanced General Math, Used by Ninth Grade General Mathematics Classes. (Colorado State College, 1966.) Dissertation Abstracts 27A: 2440-2441; February 1967.

The group using the traditional textbook and calculators for a full year gained significantly more than the group using the traditional textbook alone or the modern textbook with calculators, but only on arithmetic fundamentals achievement.

Fiske, Edward B. Educators Feel that Calculators Have Both Pluses and Minuses. The New York Times, Section IV, p. 7; January 5, 1975.

The views of proponents and opponents for the use of calculators in the classroom are presented.

Free, J. R. P. S. Buyers Guide to Under $100 Electronic Calculators. Popular Science 202: 86-88, 156; March 1973.

Several types of calculators and their distinctive features are described.

Free, J. R. Those Incredible New Scientific Pocket Calculators. Popular Science 204: 124-125; April 1975.

Ways in which the calculator can be used are discussed.

Free, John R. Now There's a Personal Calculator for Every Purse and Purpose. Popular Science 206: 78-81; February 1975.

Features and functions for 37 models are tabulated.

Frye, J. F. Versatile Pocket Calculators. Electronics 1: 58-61; May 1972.

Some of the applications for calculators are discussed.

Frye, J. F. Buying and Using a Pocket Calculator. Popular Electronics 5: 62-64; May 1974.

Some common-sense things to look for when buying an electronic calculator are given. Some algorithms are also presented, for use with the less expensive calculators which do not have all capabilities built in.

Frye, John T. Selecting a Calculator. Popular Electronics 8: 94-96; December 1975.

Who will use the calculator and for what purpose, how much math the user has and/or will study, and how much the buyer wants to pay should be considered when purchasing a calculator.

Gardner, Martin. The Magic Calculator. New York Times Magazine, p. 71, January 18, 1976.

Four tricks with a calculator are presented, with how and why they work indicated.

Gaslin, William Lee. A Comparison of Achievement and Attitudes of Students Using Conventional or Calculator-Based Algorithms for Operations on Positive Rational Numbers in Ninth-Grade General Mathematics. Dissertation Abstracts International 33A: 2217; November 1972.

Gaslin, William L. A Comparison of Achievement and Attitudes of Students Using Conventional or Calculator-Based Algorithms for Operations on Positive Rational Numbers in Ninth-Grade General Mathematics. Journal for Research in Mathematics Education 6: 95-108; March 1975.

Use of units in which fractional numbers were converted to decimals and examples then solved on a calculator was found to be a "viable alternative" to use of conventional textbooks (including fractions) with or without a calculator, for low-ability or low-achieving students.

Gibb, E. Glenadine. Calculators in the Classroom. Today's Education 64: 42-44; November-December 1975.

Ways in which the calculator can be used are discussed.

Gibb, Glenadine. My Child Wants a Calculator! NCTM Newsletter 12: 1; December 1975.

Some suggestions for selecting and using a calculator are given.

Gilbert, Jack. Advanced Applications for Pocket Calculators. Blue Ridge Summit, Pennsylvania: Tab Books, 1975.

Mathematical and scientific problems and examples for all types of calculators are included.

Gronbach, Rita. Readers' Dialogue. Arithmetic Teacher 22: 659-660; December 1975.

A letter to the editor commenting on Stultz' article.

Grosswirth, Marvin. Calculators in the Classroom. Datamation 95: 90-91, 95; March 1975.

The pros and cons for the use of hand-held calculators in the classroom are given. Summaries of some experimental work using calculators in the classroom are presented, and the need for more research in this area is stressed.

Gwynne, Peter. New York View: Calculator Boom. New Scientist 65: 231-232; January 23, 1975.

The question of whether school children should be allowed to use calculators in the classroom is said to be at the root of a debate "raging" throughout the U.S. Proponents of calculators

argue that their use for calculations can free time to explore the basic concepts of mathematics more deeply, while opponents insist that their use will lead to a generation of mathematical illiterates. As more students acquire calculators, pressure to allow their use on examinations is increasing. Several schools routinely use calculators, and children enjoy using them. Their use offers the opportunity to do more complex problems, but also provides the tempatation to sell students arithmetic without understanding.

Hale, David S. Using Electronic Calculators. Mathematics Teaching 67: 20-21; June 1974.

Two examples of classroom use of the calculator are described, one with a fifth-year class computing the volumes of boxes which could be constructed from a piece of metal, and one with a third-year class testing hypotheses concerning volumes of similar solids.

Hannon, Herbert. The $59.95 Electronic Wonder: Its Implications for the Arithmetic Classroom. Mathematics in Michigan, pp. 3-6; November 1973.

Various implications of the use of calculators are discussed.

Hawthorne, Frank S. Hand-held Calculator: Help or Hindrance? Arithmetic Teacher 20: 671-672; December 1973.

Few curricular changes will be necessitated by the advent of the calculator. Elementary schools already emphasize an understanding of concepts and a meaningful approach to algorithms. It was partly anticipation of inexpensive calculators that impelled curriculum designers to decide that emphasis on drill was unwise. While calculators will produce no grand changes in the curriculum, calculators offer many advantages in the elementary school. Students can check their work, and laborious computation can be eliminated from problem solving activities. As with any teaching aid, they can be misused.

Hawthorne, Frank S. and Sullivan, John J. Using Hand-held Calculators in Sixth-Grade Mathematics Lessons. New York State Mathematics Teachers' Journal 25: 29-31; January 1975.

This is a report on the study by Barrett and Keefe, involving two sixth-grade classes. A posttest at the end of the year indicated that the students using calculators scored higher on tests of concepts and computation than a non-calculator group, but not as high on problem-solving tests.

Heilman, Carl. More References to Calculators. Pennsylvania Council of Teachers of Mathematics Newsletter 13: 12-14; Spring 1975.

Recent articles concerning electronic calculators are listed with brief descriptions, to call them to the attention of mathematics teachers in Pennsylvania.

Higgins, Jon L. Mathematics Programs Are Changing. *Education Digest* 40: 56-58; December 1974. (Reprint from NASSP *Curriculum Report*.)

Inexpensive pocket calculators will change both the kinds of computational skills taught and the manner in which they are taught. Knowing when operations are used, checking errors, and estimation will become the new fundamental skills. Programmable calculators will soon become commonplace, and emphasis will be placed on the unity of mathematics and the common procedures of mathematical thought.

Hoffman, Ruth I. Don't Knock the Small Calculator -- Use It! *Instructor* 85: 149-150; August/September 1975.

Some "scattered examples" of using hand-held calculators to explore mathematical ideas are presented.

Hohlfeld, Joseph Francis. Effectiveness of an Immediate Feedback Device for Learning Basic Multiplication Facts. (Indiana University, 1973.) *Dissertation Abstracts International* 34A: 4563; February 1974.

The effectivness of an electronic calculator, programmed as an immediate feedback device, was compared with the effectiveness of pencil-and-paper exercises without immediate feedback for the learning of the 100 basic multiplication combinations. Twelve students in each of seven fifth-grade classes were identified as low achievers and randomly assigned to treatment. Significant differences favored the electronic calculator practice group over the pencil-and-paper practice group on both acquisition and short-term retention, but not on long-term retention (one month or three-and-one-half months retention periods).

Huff, D. Teach Your Pocket Calculator New Tricks to Make Like Simpler. *Popular Science* 205: 96-98; December 1974.

Interesting things to do with a hand calculator are presented.

Huff, Darrell. How to Have Fun with Your Pocket Calculator. *Popular Science* 208: 90-91, 152; February 1976.

Activities and games for the calculator are presented.

Hunter, William L. *Getting the Most Out of Your Electronic Calculator*. Blue Ridge Summit, Pennsylvania: Tab Books, 1974.

This book describes some basic problems that the layman can solve using a pocket calculator. Working with basic mathematics, grocery shopping, computing simple interest, and tax preparation are some of the topics included.

Jefimenko, Oleg. How to Entertain with Your Pocket Calculator. Star City, West Virginia: Electret Scientific Co., 1975.

Mathematical diversions are presented.

Jesson, David and Kurley, Frank. Specifications for Electronic Calculators. Mathematics Teaching 70: 42-43; March 1975.

A basic electronic calculator for the mathematics classroom should have the following features, listed in order of preference: (1) natural-order arithmetic, (2) floating point, (3) underflow, (4) constant key to operate on all four operations, (5) eight-digit display, (6) fingertip-size keys, (7) rechargeable batteries with alternative plug-in operation, and (8) clear-entry key. The prospective buyer can check for each of these features using simple tests. The educational market is a clear growth area for calculator manufacturers, and should not hesitate to demand what is needed.

Johnson, Randall Erland. The Effect of Activity Oriented Lessons on the Achievement and Attitudes of Seventh Grade Students in Mathematics. (University of Minnesota, 1970.) Dissertation Abstracts International 32A: 305; July 1971

Activity-oriented instruction, including one treatment in which calculators were used, did not appear to be more effective than instruction with little or no emphasis on activities, for units in number theory, geometry and measurement, and rational numbers.

Jones, C. D. Pocket Math. National Elementary Principal 53: 56-57; January 1974.

Three children (aged 6, 7, and 10), who used a pocket calculator for one-and-one-half weeks, learned to use the machine in self-initiated activity. However, it took a while for them to trust it.

Judd, Wallace. Games, Tricks, and Puzzles for a Hand Calculator. Menlo Park, California: Dymax, 1974.

This book is a source of recreational ideas for use with the calculator, with sections on each of the topics of the title. Included also is a section on the internal workings of a calculator.

Judd, Wallace. A New Case for the Calculator. Learning 3: 41-48; March 1975.

A visit to a classroom in which calculators are used reveals that children work well with them. As calculators become increasingly available, a change in classroom emphases will occur; drill will be de-emphasized in favor of problem-solving activities. Students will need to have firm concepts of place value. The calculator will never replace mathematical understanding, but it is here to stay. Considerations for selecting calculators and selected games for use with calculators are described.

Kelley, J. L. and Lansing, Ira. Interim Report on Project EQUIP. Berkeley, California, July 1975.

This document reports on Berkeley's mini-calculator program for teaching mathematics to low achievers in junior high school classes. Data on four experimental and four control classes are presented.

Keough, John J. and Burke, Gerald W. *Utilizing an Electronic Calculator to Facilitate Instruction in Mathematics in the 11th and 12th Grades. Final Report*. July 1969. ERIC: ED 037 345. 60 pages.

The group using calculators achieved significantly more on a standardized test than did a group not using them.

Kessner, Arthur. Exploring Calculators in Primary Mathematics. Berkeley, California: Regents of the University of California, 1975.

A report on a study with primary grade children is presented.

Ladd, Norman Elmer. The Effects of Electronic Calculators on Attitude and Achievement of Ninth Grade Low Achievers in Mathematics. (Southern Illinois University, 1973.) *Dissertation Abstracts International* 34A: 5589; March 1974.

Two-hundred-one low achievers were randomly scheduled into one of five control sections or one of five experimental sections. All groups followed the same lesson sequence, with control groups using only paper-and-pencil for all calculations and experimental sections using electronic calculators. Significant differences were found on both attitude and achievement tests from pre- to post-treatment for both groups, but no significant differences in posttest mean scores were found between groups.

Lecker, Lorna. Mini-Calculators Help Students Enjoy Math. *The Burlington (Vermont) Free Press*, p. 19, June 20, 1972.

A report on the positive influence that two electronic calculators had on the students in a junior high mathematics program is given.

Lesjack, J. J. Computation: Beat the Machine. *Grade Teacher* 87: 150, 152-153; March 1970.

A game in which teams of students compete with each other using an adding machine can be used to practice and enhance basic skills.

Lewis, J. A Further Review of Scientific Calculators. School Science Review 56: 625-628; Number 196, March 1975.

Eighteen calculators available in Britain are analyzed.

Lewis, Philip. Minicalculators Have Maxi-impact. Nation's Schools 93: 60, 62; May 1974.

The pocket calculator is not only cheaper and more convenient than the calculators of a few years ago; it can also do more. It may revolutionize education by saving students and teachers a great deal of computational time. Checking answers will become routine. Students may even use mathematics outside of class. When buying calculators, several optional features should be sought: built-in battery recharger, battery drain indicator, display cut-off, decimal capability, numeric display indicator, high impact plastic housing, and warranty. An increasing number of models and extra features are available.

Lindsay, Robert L. Black Box Numeracy. Mathematics in School 4: 26-28; November 1975.

The effect of the use of hand-held calculators on the arithmetic curriculum is explored.

Longstaff, F. R. et al. Desk Calculators in the Mathematics Classroom. June 1968. ERIC: ED 029 498. 11 pages.

This study was designed to test the use of calculators with two groups of ninth graders and one group of fifth graders. The findings were equivocal, concerning the effect of calculators on students' performance, self-confidence, and attitudes toward mathematics. Teacher enthusiasm for calculator use was unrelated to student performance. Teacher enthusiasm was highest in classes of low-average IQ. While some teachers felt calculators interfered with their daily operations, others felt that the productivity of students increased, especially among those previously incapable of producing. Classroom behavior problems were eased.

Machlowitz, Eleanore. Electronic Calculators -- Friend or Foe of Instruction? Mathematics Teacher 69: 104-106; February 1976.

Some suggestions for using calculators are presented.

Maeroff, Gene I. Calculators Termed Good Tool for Pupils. New York Times, December 24, 1975.

The NCTM position on calculators is reported.

Mastbaum, Sol. A Study of the Relative Effectiveness of Electric Calculators or Computational Skills Kits in the Teaching of Mathematics. (University of Minnesota, 1969.) Dissertation Abstracts International 30A: 2422-2423; December 1969.

The calculator, when used as a teaching aid with slow learners in mathematics in the seventh and eighth grades, did not significantly improve attitude, increase mathematical achievement, or increase non-calculator computational skill, mastery of mathematical concepts, or ability to solve mathematical problems. However, the students did at least as well in all areas as those students not using calculators.

McCluggage, D. Calculators: Mini-Calculators. American Home 76: 25-26; April 1973.

Uses of hand-held calculators are discussed in this article.

McShane, Jane. Electric Calculators; Business Education: 7718.06. (Quinmester Course, Dade County Public Schools, Miami, Florida.) 1972. ERIC: ED 097 570. 30 pages.

A course was developed to instruct business students in the use of mechanical and electronic printing calculators, and electronic display calculators. Performance objectives, course content, suggested activities, evaluative instruments, and resource materials are described.

Miller, R. Make a Giant Step Forward with the Latest in Electronics Technology: The Mini-calculator. Man/Society/Technology 33: 5-6; September 1973.

The potential of the use of calculators is discussed.

Millikin, G. and Siegel, D. Kit for Teaching Calculating and Computing Devices. Teaching Exceptional Children 3: 17-22; Fall 1970.

A kit was designed to introduce gifted students to basic computer activities. The kit included an abacus, slide rule, desk calculator, punch-card equipment, and an electronic computer, as well as books. A series of objectives and activities is outlined.

Mims, F. $20 Mini-computer Revolution. Science Digest 75: 41-45; May 1974.

Due to rapidly advancing technology, especially the development of silicon large-scale integration chips, the price of vestpocket calculators is going down, while the capability of these devices continues to increase. Calculator manufacturers are now working on streamlining the pocket models even more, and perfecting keyboard design. Advanced scientific and business pocket calculators are currently on the market, as are models which perform conversions between the metric and English systems of measurement. A somewhat large calculator with printout capabilities is available, and programmable calculators are being developed. In the future, calculators with large memories will be available. One pocket-sized machine will serve as a combination computer, telephone directory, notepad, calendar, and dictionary.

Mims, F. Calculators: From the Abacus to the Electronic Calculator. *Radio-Electronics* 43: 51-54; December 1972.

The development of calculating machines is traced.

Moss, Rosalind. Using Electronic Calculators. *Mathematics Teaching* 67: 20-21; June 1974.

Suggestions for using calculators in the classroom are discussed.

Mullish, Henry. *How to Get the Most Out of Your Pocket Calculator*. New York: Collier Books, 1974.

This book describes how pocket calculators can be used by the average person. Such things as determining gas consumption of one's car, balancing a checkbook, shopping for best buys, computing interest, and similar topics are covered. A section on higher-priced calculators is included along with sample problems and answers.

North, Roger. Using a Hand-held Electronic Calculator. *Mathematics in School* 4: 22-23; March 1975.

An explanation of the features and operating procedures for one British calculator are given, with illustrations for solving various types of problems.

Offenheiser, Marilyn. U. S. Homes in on Calculators. *Electronics* 45: 69-71; September 25, 1972.

Challenging Japanese domination of the business calculator sales, Americans are exploiting technology, not merely exporting it. Reduced prices for U. S. calculators are upstaging cheap Japanese labor.

Osborne, J. M. The Pocket Calculator in School Physics. *Physics Education* 9: 414-419; September 1974.

The significance of the simple, inexpensive type of calculator as a tool in the classroom is discussed, as well as the contribution of the more sophisticated type to the school science department.

Parks, Terry E. Minicalculators: Opportunity or Dilemma? *Bulletin of the Kansas Association of Teachers of Mathematics* 49: 18-21; April 1975.

The phenomena which will have the most profound effect on mathematics education in the 1970s are the changeover to the metric system and the rapidly increasing use of hand calculators.

Reportedly 30 to 50 per cent of all secondary students in Kansas City have access to calculators, yet relatively little has been written about their use in schools. Research on the effects of such use has produced mixed results. The decision to use or not use calculators in the classroom must depend on many factors. Calculators can be used functionally as an eraser or desk, or pedagogically like textbooks and flashcards. Used in either way they will necessarily cause curricular changes; they deal in decimals, not fractions, and handle negative numbers as easily as positive. The major benefit of calculator use will be in the area of motivation.

Pendleton, Deedee. Calculators in the Classroom. Science News 107: 175, 181; March 15, 1975.

Both the manufacturers of electronic calculators and "progressive" educators are anxious to see a calculator in every classroom. Although opponents claim that students using them won't know how to count when the batteries die, instructors using them say calculators increase students' interest and enable them to solve more interesting problems.

Priest, E. C. Additional Functions for Your Pocket Calculator. Popular Electronics 14: 64; October 1973.

By knowing a few simple procedures, one can make a simple four-function calculator perform many functions. Positive integral exponentials, reciprocals, and square roots are easily computed by the described methods.

Quadling, Douglas. A Nation of Button Pushers? Mathematics in School 4: 23; May 1975.

The use of electronic calculators in the British classroom is examined. Concern center on the extent of used, with basic questions about use on tests, fairness to those who cannot afford them, and the danger of them becoming a "crutch".

Quinn, Donald R. Yes or No? Calculators in the Classroom. NASSP Bulletin 60: 77-80; January 1976.

Reasons for using or not using calculators are discussed.

Quinn, Mildred Louisa. Accounting Class Failures and Arithmetic Deficiencies. Business Education Forum 28: 30-31; March 1974.

A common cause of failure in accounting, student inability to compute mentally, could be resolved if a calculator were available to all students.

Riden, Chuck. Less Than Ten on a Calculator. School Science and Mathematics 75: 529-531; October 1975.

A method to check addition and subtraction in any number base less than ten, using a simple adding machine or a calculator, is given.

Roberts, Edward M. Fingertip Math. Dallas: Texas Instruments, 1974.

This book explains how to use a hand-held calculator effectively.

Rogers, J. T. The Calculator Book: Fun and Games with Your Pocket Calculator. New York: Random House, 1975.

This book presents many activities for the calculator.

Schafer, Pauline; Bell, Max S.; and Crown, Warren D. Calculators in Some Fifth-Grade Classrooms: A Preliminary Look. Elementary School Journal 76: 27-31; October 1975.

Students in the calculator group scored significantly higher on calculator examples, while no differences were found on noncalculator examples between calculator and noncalculator groups.

Schilt, H. Use of Calculators in Swiss Schools. Arithmetic Teacher 9: 129; March 1962.

The use of hand-cranked Curta calculators is described.

Schott, A. F. Adventure in Arithmetic. Educational Screen 34: 65-67; 1955.

A study with students in grades 4 through 9 is reported; the groups using calculators achieved higher than groups not using calculators.

Shapiro, Donald. How To Add Functions to Simple Hand Calculators. Popular Electronics 8: 38; September 1975.

Instructions for modifying the calculator are provided.

Shaw, Bob. Equipment for the Mathematics Room. Mathematics in School 2: 30-31, November 1973.

Among other materials, eight permanently fixed but readily available calculators are suggested.

Shea, James Francis. The Effects on Achievement and Attitude Among Fourth Grade Students Using Calculator Flow-Charting Instruction vs. Conventional Instruction in Arithmetic. (New York University, 1973.) Dissertation Abstracts International 34A: 7499; June 1974.

The group having calculator instruction had significantly higher scores than a group not using calculators on computation but not other tests or an attitude measure.

Shumway, Richard J. Mathematical Problem Solving, Children (9-12 Years), Teachers, Hand Calculators, and Research. Unpublished manuscript, prepared for George Springer, Indiana Problem Solving Project; September 1974.

Psychological and mathematical considerations in regard to problem solving are discussed, with the use of the hand-held calculator one of many points.

Skoll, Pearl. *Coping with the Calculator*. Northridge, California: Pearl Skoll, 1975.

Activities for use with calculators are presented.

Smith, J. M. *Scientific Analysis on the Pocket Calculator*. New York: Wiley, 1975.

Suggestions for more advanced uses of the calculator are presented.

Spencer, JoAnn Nora. Using the Hand-held Calculator in Intermediate Grade Arithmetic Instruction. (Lehigh University, 1974.) *Dissertation Abstracts International* 35A: 7048-7049; May 1975.

The calculator group scored significantly better than the non-calculator group on the reasoning test in grade 5 and on the computation test and total test in grade 6.

Stocks, Sister Tina Marie. The Development of an Instructional System Which Incorporates the Use of an Electric Desk Calculator as an Aid to Teaching the Concept of Long Division to Educable Mentally Retarded Adolescents. (Columbia University, 1972.) *Dissertation Abstracts International* 33A: 1049-1050; September 1972.

All students demonstrated an improvement in scores between pre- and posttest; however, no tests of significance were made. A positive change in attitude was also found.

Stultz, Lowell. Electronic Calculators in the Classroom. *Arithmetic Teacher* 22: 135-138; February 1975.

Several applications of calculators at various elementary grade levels are suggested, with some punch/display sequences illustrated.

Thiagarajan, Sivasailam. Calculators are "In" and Cheap: Here are Four Games You Can Play Using the Least Expensive. *Simulation/Gaming/News* 2: 10-13; January 1975.

The games may be appropriate for older students.

Triggs, E. The Value of a Desk Calculating Machine in Primary School Mathematics. *Educational Research* 9: 71-73; November 1966.

Two groups of students, matched for ability, were found to make significant gains whether or not they used the calculator.

Van Atta, Frank. Calculators in the Classroom. *Arithmetic Teacher* 14: 650-651; December 1967.

Many problems cannot be done by the pupil alone, but can be handled by the pupil-plus-computer combination. Two such problems involve exponents and the Pythagorean theorem. The facility to do many computations enables students to get a better feel for rational and irrational numbers and for the definition of a logarithm.

Wasson, Ruth Ann. The Medium is the Message...Or, There is a Message in the Medium. Journal of Business Education 48: 69-70; November 1972.

Experience shows that students are accepting the use of calculators because it saves them time and provides more realistic experience related to business practice.

Weaver, J. F. Materials for use in workshops on using the hand-held calculator. Madison: University of Wisconsin, 1975.

Various materials prepared by the author include a list of books and articles, questions to ask when purchasing a calculator, descriptions of various models, and items to use with children.

Weaver, J. Fred. Readers' Dialogue. Arithmetic Teacher 22: 658-659; December 1975.

A letter to the editor commenting on Stultz' article.

Weaver, J. F. Calculator-influenced Explorations in School Mathematics: Number Sentences and Sentential Transformations I, II. Project Paper 76-1. Madison: Wisconsin Research and Development Center for Cognitive Learning, January 1976.

Explorations involving the use of mini electronic calculators in connection with mathematics instruction were conducted with two fifth-grade classes, two second-grade classes, and three third-grade classes. Limited empirical data suggest that pupils encounter no consequential problems with the mechanics of using simple four-function, algebraic-logic calculators in routine contexts, and that pupils likely will elect not to use calculators in situations where their use is unnecessary or of no particular advantage. While elementary-school mathematics programs usually emphasize binary operations, project explorations have moved increasingly toward content interpretations in terms of unary operations.

Adding and Calculating Machines. Consumer Bulletin 55: 14-18; September 1972.

Features, prices, and things to look for when buying are included, with ratings of ten electric adding machines and one electronic hand-held calculator.

Calculated Boom. Newsweek 80: 84; October 2, 1972.

The increasing popularity of electronic calculators is reported.

Calculated Warfare. Time 100: 95; October 30, 1972.

A status report on calculators is given, with the price war noted.

Calculation or Computation? Is That the Question? Teacher, p. 52; March 1975.

The role of the calculator in schools is discussed.

Calculators. Consumer Guide 1975 Consumer Buying Guide. New York: New American Library, Signet edition, 1975. Pp. 340-343.

Ratings for various types of calculators are presented.

Calculators. Consumer Reports 40: 533-541; September 1975.

General-purpose calculators are analyzed.

Calculators. Times Higher Education Supplement (England), No. 214, pp. 23-27, November 28, 1975.

This issue contains a series of articles, by different authors, on various aspects of calculators, including background, uses, and projections.

Calculators in the Classroom. Time 105: 88; January 6, 1975.

An overview of some initial work with calculators in the classroom is presented. Successes have been reported in computational efficiency and motivation. Problems include fear of dependency, unfair advantage, and machine-security factors.

Calculators Plus Classrooms Equal Positive Profits. Consumer Electronics News, February 1975.

The status of calculators is discussed and conclusions of manufacturers and retailers presented.

Calculators Remotivate High School Students Towards Math. Educational Equipment and Materials, Spring 1973.

Use of desk calculators with low achievers in a San Francisco high school is described.

Calculators Slim Down in Size and Price. Business Week, p. 50: October 9, 1971.

The development of chips, reducing size and price of calculators, is discussed.

A Calculator to Fit the Pocket. Business Week, pp. 28-29; April 18, 1970.

The development of one of the first pocket-sized calculators is described.

Effectiveness of Calculators in Teaching Children Both Fundamentals and Concepts of Math. Educational Equipment and Materials, Spring 1973.

A pilot program with pupils in grades 4-6 is reported.

Electronic Brains in the Classrooms - A+ or -? U. S. News & World Report, p. 30; January 13, 1975.

Views are presented for and against the use of calculators.

Electronic Calculators. Consumer Bulletin 56: 15-19; May 1973.

Various types of calculators are discussed.

Electronic Calculators. Changing Times 27: 39-41; July 27, 1973.

Differing characteristics of calculators are considered, with a summary for twelve hand-held and eleven desk calculators.

Electronic Calculators. The Complete Buyer's Guide: Best Values '75. New York: Service Communications, Guide No. 18, 1975. Pp. 49-64.

Various calculators are evaluated.

Electronic Mini-calculators. Consumer Reports 38: 372-377; June 1973. 38: 663; November 1973.

Basic characteristics, prices, and ratings for calculators are presented.

Electronic Pocket Calculators. Consumer's Research Magazine 57: 7-12; September 1974. 58: 19; January 1975.

Characteristics and ratings of calculators are given.

Games Calculators Play. Time 103; 56; June 24, 1974.

Using the calculator for games and spelling words is discussed.

Great Calculator Debate. Nation's Schools and Colleges 1: 12, 26; December 1974.

Reasons for using or not using the calculator are discussed.

How to Pick an Electronic Calculator. Better Homes and Gardens 51: 162; April 1973.

Some features to look for when buying a hand-held calculator include floating decimal, negative function, clear key, ability to do mixed calculations, and power source.

Math Lab Matrix (Illinois State University bulletin), Spring 1976.

This issue is devoted to articles about using calculators.

The Mini-Calculator Project. Eye on Education 4: 8-9; Number 1, 1975. (Temple University College of Education.)

A large-scale investigation of the effects of the availability and use of the hand-held calculator upon the attitudes and mathematics achievement of seventh-grade students is described.

Minicalculators in Schools. Arithmetic Teacher 23: 72-74; January 1976. Mathematics Teacher 69: 92-94; January 1976.

This report from the NCTM Instructional Affairs Committee presents nine justifications for using the hand-held calculator, with some specific examples of curricular applications.

Minicalculators Steal the Consumer Show. Business Week, pp. 29-30; June 16, 1973.

The market situation is reviewed.

New Markets for Programmable Calculators. Business Week, pp. 109, 112-113; June 15, 1974.

The status and projections for sales of calculators are given.

Now -- There's a Personal Calculator for Every Purse and Purpose. Popular Science 206: 78-81, 136; February 1975.

Types of calculators are described and discussed.

Numbers Game: Games with Pocket Calculators. Newsweek 85: 66; March 17, 1975.

Several games played on calculators are described.

The Omron Express! Mountain View, California: Omron, 1972. (See ERIC: ED 079 923.)

This 30-page comic book is designed to introduce elementary-school children to electronic calculators.

One Neat Little Way To Get Your Schools Ready for the Electronic Teaching Marvels Coming at You. American School Board Journal 160: 46-47; September 1973.

Some plans for providing electrical and communication outlets in an open-plan school are described.

Overview and Analysis of School Mathematics Grades K-12. Washington: Conference Board of the Mathematical Sciences, National Advisory Committee on Mathematical Education (NACOME), November 1975.

In the report, references to the use of calculators are made at many points, including pages 24-25, 34, 40-43, 138, 141, and 145.

Pocketing the Profits. Dun's 100: 89-90; September 1972.

The current status and future projection for the manufacture of calculators is given.

Quiet Show Worries Calculator Makers. Business Week, pp. 109 ff.; June 15, 1974.

The slowdown of interest in calculator sales is described.

Sales Boom: Prices Plummet in Pocket Calculator Biz. Purchasing, November 19, 1974.

This is a brief report on the status of sales in calculators.

Scientific Calculators. Consumer Reports 41: 86-87; February 1976.

This update on the September 1975 CR report includes analyses of four calculators.

Teachers Approve of Mini-Calculator Use in Classrooms. Math's Alive, volume IV, No. 10. Philadelphia, Pennsylvania: Office of Curriculum and Instruction, The School District of Philadelphia, June 1975.

Results of a questionnaire sent to all secondary mathematics teachers in Philadelphia public schools are presented and discussed.

Those Little Calculators. Bulletin of the Council for Basic Education, p. 5; January 1975.

The impact of calculators is briefly discussed.

Tricks with Calculators. Machine Design, pp. 170 ff.; June 13, 1974.

Some ways to use calculators are discussed.

Utilizing an Electronic Calculator. A Manual for Planning and Development. January 1970. ERIC: ED 038 030. 53 pages.

An electronic calculator was used to aid mathematics instruction in grades 11 and 12. Requirements for implementing the program are described and illustrated by experiences at the pilot school. A curriculum guide is given for the inservice teacher training program, and suggestions for using the calculator in a mathematics program are included.

What to Look For in an Electronic Calculator. Christian Science Monitor, December 10, 1974.

Some guidelines are given for those purchasing a calculator.

Where Do You Stand? Computational Skill Is Passe. Mathematics Teacher 67: 485-488; October 1974.

A survey of teachers, mathematicians, and laymen is reported, with seven questions and percentage of responses noted.

World Electronics Markets: Mounting a Push Toward a Slow Modest Recovery. Electronics 49: 83-106; January 8, 1976.

Projections on sales of calculators are included amid much other data.

1984: A Calculator on Every Wrist and in Every School Desk, Too. Nation's Schools and Colleges, p. 13; December 1974.

What will be available for calculators is predicted by Mullish. He expects many of the sophisticated capabilities now on expensive models, such as programming capacity, to be found on most standard models in a few years.

Appendix B:

1. Summarized Responses from Manufacturers and Distributors

The questionnaire (pages B-2 to B-4) was sent to 39 manufacturers and distributors; responses were received from only 7. This information is collated on pages B-5 to B-11). In an attempt to secure further information, some were contacted by telephone. In addition a "blind" request for advertising information was sent, and information on specifications was collated from these materials. This information is presented on pages B-12 to B-16).

Especially evident (amid the general lack of response) was the helpfulness of the representatives of two companies - Hewlett-Packard and Novus - about the role of hand-held calculators from their perspective.

Survey on Calculators

Sample: Calculator Manufacturers

1. Please fill in information on models of hand-held calculators that you manufacture which you consider appropriate for use in elementary and/or secondary schools, and check the features of each:

	Model number									
	Current retail cost									
	Rechargable?									
	Adapter cost									
FEATURES (check)	+, -, x, ÷									
	$\sqrt{\ }$									
	scientific notation									
	floating decimal									
	log (base 10)									
	ln (base e)									
	10^x									
	e^x									
	x^y									
	sin									
	cos									
	tan									
	arcsin/cos/tan									
	hyperbolic sin/cos/tan									
	n!									
	$x \rightleftarrows y$									
	stacking storage									
	independent storage									
	programmable?									
other special functions (list)										
	length of guarantee									
	predicted cost: 3 yrs.									
	5 yrs.									
	10 yrs.									

2. Do you advertise hand-held calculators as being useful in doing school work?

___ Yes

___ No

3. Do you employ staff specifically to help educators/schools use hand-held calculators?

___ Yes

___ No

4. Do you produce educational materials to be used with hand-held calculators?

___ Yes (We would appreciate samples of these materials.)

___ No

5. Approximately what percentage of your sales of hand-held calculators is made to elementary and/or secondary schools?

___ a. 0%

___ b. 1-25%

___ c. 26-50%

___ d. 51-75%

___ e. 76-100%

6. What do you estimate to be the total sales (to date) in the United States of hand-held calculators?

	of all manufacturers	of your company
a. number	___	___
b. $ amount	___	___

7. What do you project for sales of hand-held calculators in the United States?

	of all manufacturers		of your company	
	number	$ amount	number	$ amount
a. for the rest of 1975	___	___	___	___
b. for 1976	___	___	___	___
c. for 1975-77	___	___	___	___
d. for 1975-80	___	___	___	___
e. for 1975-85	___	___	___	___

8. Approximately what percentage of sales of hand-held calculators do you estimate will be made to elementary and/or secondary schools?

	of all manufacturers		of your company	
	number	$ amount	number	$ amount
a. for the rest of 1975				
b. for 1976				
c. for 1975-77				
d. for 1975-80				
e. for 1975-85				

9. What types of consumers are buying hand-held calculators?

Type	Estimated % of market
a. Schools (K-12)	
b. Individuals for general home uses	
c. Individuals for work-related uses	
d. Other	

10. What do you predict for the future technology in this field?

1. Please fill in information on models of hand-held calculators that you manufacture which you consider appropriate for use in elementary and/or secondary schools, and check the features of each:

← Adler →

	Model number	81C	81S	88T	108T	82M				
	Current retail cost	34.95	39.95	79.95	119.95	99.95				
	Rechargable?	✓	✓	✓	✓	✓				
	Adapter cost	11.95	11.95	11.95	incl.	11.95				
FEATURES (check)	+, −, x, ÷	✓	✓	✓	✓	✓				
	$\sqrt{}$		✓	✓	✓					
	scientific notation				✓					
	floating decimal	✓	✓	✓	✓					
	log (base 10)			✓	✓					
	ln (base e)			✓	✓					
	10^x				✓					
	e^x			✓	✓					
	x^y				✓					
	sin			✓	✓					
	cos			✓	✓					
	tan			✓	✓					
	arcsin/cos/tan			✓	✓					
	hyperbolic sin/cos/tan				✓					
	n!				✓					
	$x \rightleftarrows y$			✓	✓					
	stacking storage									
	independent storage	✓	✓	✓	✓	✓				
	programmable?									
other special functions (list)										
	length of guarantee	1 yr.	1 yr.	1 yr.	1 yr.	1 yr.				
	predicted cost: 3 yrs.									
	5 yrs.									
	10 yrs.									

OSU/MNS-1

1. Please fill in information on models of hand-held calculators that you manufacture which you consider appropriate for use in elementary and/or secondary schools, and check the features of each:

		Canon								
	Model number	LE84	LE85	LD80	LD81	LE81	F5	F7		
	Current retail cost	14.95	19.95	19.95	18.95	59.95	79.95	169.95		
	Rechargable?									
	Adapter cost	5.00	5.00	5.00	5.00	10.00	10.00	10.00		
FEATURES (check)	+, -, x, ÷	✓	✓	✓	✓	✓	✓	✓		
	$\sqrt{\ }$		✓		✓	✓	✓	✓		
	scientific notation							✓		
	floating decimal	✓	✓	✓	✓	✓	✓	✓		
	log (base 10)						✓	✓		
	ln (base e)						✓	✓		
	10^x						✓	✓		
	e^x						✓	✓		
	x^y						✓	✓		
	sin						✓	✓		
	cos						✓	✓		
	tan						✓	✓		
	arcsin/cos/tan						✓	✓		
	hyperbolic sin/cos/tan							✓		
	n!									
	$x \rightleftarrows y$							✓		
	stacking storage							✓		
	independent storage							✓		
	programmable?									
other special functions (list)										
	length of guarantee	1 yr	1 yr	1 yr	1 yr	1 yr	1 yr	1 yr.		
	predicted cost: 3 yrs.									
	5 yrs.									
	10 yrs.									

1. Please fill in information on models of hand-held calculators that you manufacture which you consider appropriate for use in elementary and/or secondary schools, and check the features of each:

← Hewlett- Packard →

	Model number	21	25	35	70	45	55	65		
	Current retail cost	125.00	195.00	195.00	275.00	245.00	395.00	795.00		
	Rechargable?	✓	✓	✓	✓	✓	✓	✓		
	Adapter cost									
FEATURES (check)	+, -, x, ÷	✓	✓	✓	✓	✓	✓	✓		
	$\sqrt{}$	✓	✓	✓		✓	✓	✓		
	scientific notation	✓	✓	✓	✓	✓	✓	✓		
	floating decimal	✓	✓	✓	✓	✓	✓	✓		
	log (base 10)	✓	✓	✓		✓	✓	✓		
	ln (base e)	✓	✓	✓		✓	✓	✓		
	10^x	✓	✓	✓		✓	✓	✓		
	e^x	✓	✓	✓		✓	✓	✓		
	x^y	✓	✓	✓	✓	✓	✓	✓		
	sin	✓	✓	✓		✓	✓	✓		
	cos	✓	✓	✓		✓	✓	✓		
	tan	✓	✓	✓		✓	✓	✓		
	arcsin/cos/tan	✓	✓	✓		✓	✓	✓		
	hyperbolic sin/cos/tan									
	n!		✓			✓	✓	✓		
	$x \rightleftarrows y$	✓	✓	✓	✓	✓	✓	✓		
	stacking storage	✓	✓	✓	✓	✓	✓	✓		
	independent storage						✓	✓		
	programmable?						✓	✓		
other special functions (list)										
	length of guarantee	1 yrs.	1 yrs.	1 yrs.	1 yrs.	1 yrs.	1 yrs.	1 yrs.		
	predicted cost: 3 yrs.									
	5 yrs.									
	10 yrs.									

OSU/MNS-1

1. Please fill in information on models of hand-held calculators that you manufacture which you consider appropriate for use in elementary and/or secondary schools, and check the features of each:

		← Novus →								
	Model number	650	850	824T	824R	4510*	4520*	Quiz Kid	Whiz Kid	
	Current retail cost	9.95	12.95	39.95	49.95	49.95	79.95	17.95	17.95	
	Rechargable?		✓		✓					
	Adapter cost		5.95	5.95		5.95	5.95		5.95	
FEATURES (check)	+, -, x, ÷	✓	✓	✓	✓	✓	✓	✓	✓	
	$\sqrt{\quad}$					✓	✓			
	scientific notation						✓			
	floating decimal		✓	✓	✓	✓	✓	✓	✓	
	log (base 10)					✓	✓			
	ln (base e)					✓	✓			
	10^x						✓			
	e^x					✓	✓			
	x^y					✓	✓			
	sin					✓	✓			
	cos					✓	✓			
	tan					✓	✓			
	arcsin/cos/tan					✓	✓			
	hyperbolic sin/cos/tan									
	n!									
	$x \rightleftarrows y$					✓	✓			
	stacking storage					✓	✓			
	independent storage			✓	✓	✓	✓			
	programmable?					*	*			
other special functions (list)	fully accumulating memory			✓	✓					
	% Key			✓	✓					
	%±			✓	✓					
	change sign			✓	✓					
	Exchange			✓	✓					
	length of guarantee	1 yr.	1 yr.	1 yr.	1 yr.	1 yr.	1 yr.	90 day	90 day	
	predicted cost: 3 yrs.		10.00	30.00	35.00	25.00	50.00			
	5 yrs.									
	10 yrs.									

OSU/MNS-1

* Programmable models are available
These models also have π and 1/x Keys

1. Please fill in information on models of hand-held calculators that you manufacture which you consider appropriate for use in elementary and/or secondary schools, and check the features of each:

		Victor		← Casio →				← Compucorp →		
	Model number	225		CP801C	FX11	FX15		324	344	326
	Current retail cost	129.00		19.95	49.95	64.95		600.00	600.00	900.00
	Rechargable?	✓						✓	✓	✓
	Adapter cost				2.50	2.50		incl.	incl.	incl.
FEATURES (check)	+, -, x, ÷	✓		✓	✓	✓		✓	✓	✓
	$\sqrt{}$				✓	✓		✓	✓	✓
	scientific notation					✓		✓	✓	✓
	floating decimal	*		✓	✓	✓		✓	✓	✓
	log (base 10)				✓	✓		✓	✓	✓
	ln (base e)				✓	✓		✓	✓	✓
	10^x				✓	✓		✓	✓	✓
	e^x				✓	✓		✓	✓	✓
	x^y				✓	✓		✓	✓	✓
	sin				✓	✓		✓	✓	✓
	cos				✓	✓		✓	✓	✓
	tan				✓	✓		✓	✓	✓
	arcsin/cos/tan					✓		✓	✓	✓
	hyperbolic sin/cos/tan									
	n!								✓	
	$x \rightleftarrows y$									
	stacking storage			✓	✓	✓				
	independent storage	✓				✓		✓	✓	✓
	programmable?							✓	✓	✓
other special functions (list)	% Key			✓						
	π				✓	✓				
	1/x					✓				
	sexagesimal conversion				✓	✓				
	length of guarantee	1 yr.		1 yr.	1 yr.	1 yr.		6 mo.	6 mo.	6 mo.
	predicted cost: 3 yrs.									
	5 yrs.									
	10 yrs.									

OSU/MNS-1

* floating decimal for entries, fixed out

2. *Do you advertise hand-held calculators as being useful in doing school work?*

Yes: 6
No: 1

3. *Do you employ staff specifically to help educators/schools use hand-held calculators?*

Yes: 3
No: 4

4. *Do you produce educational materials to be used with hand-held calculators?*

Yes: 2
No: 4
Now producing material: 1

5. *Approximately what percentage of your sales of hand-held calculators is made to elementary and/or secondary schools?*

0 - 5%: 1
l - 25%: 1
26 - 50%: 1

No response: 4

6. *What do you estimate to be the total sales (to date) in the United States of hand-held calculators?*

	of all manufacturers	*of your company*
a. number	30-35 million - 1 15-17 million - 1 no response - 5	No response - 7
b. $ amount	$250-350 million - 1 $530 million - 1 no response - 5	No response - 7

7. *What do you project for sales of hand-held calculators in the United States?*

	of all manufacturers		*of your company*	
	number	*$ amount*	*number*	*$ amount*
a. for the rest of 1975	10 million	$250 million	No response - 7	
b. for 1976	18-20 million	$550 million		
c. for 1975-1977	20-23 million	$590 million		
d. for 1975-1980	23-25 million	$600 million		
e. for 1975-1985	25-28 million	$630 million		
Total by 1980	100 million	$2.5-5 billion		

No response - 5

8. *Approximately what percentage of sales of hand-held calculators do you estimate will be made to elementary and/or secondary schools?*

	of all manufacturers		*of your company*	
	number	*$ amount*	*number*	*$ amount*
a. for the rest of 1975	10%	(Only 1 response; no dollars named)	No response - 7	
b. for 1976	10%			
c. for 1975-77	10%			
d. for 1975-80	12%			
e. for 1975-85				

9. *What types of consumers are buying hand-held calculators?*

Type	*Estimated % of market*	
a. Schools (K-12)	7%, 10%, 10%	No response - 4
b. Individuals for general home uses	25%, 30%, 60%	No response - 4
c. Individuals for work-related uses	22%, 50%, 50%	No response - 4
d. Other	1%, 10%, 15%	No response - 4

10. *What do you predict for the future technology in this field?*

- Specialized function units such as metric converter. Programming capability also.
- Much more power in programmable machines for lower prices as technology and competition progress. More speciality models for specific requirements. These are called dedicated machines.
- It will be changed.
- Unlimited.

No response - 3

Additional Sources of Information on Calculator Availability and Features

A. Blind Requests for Advertising

Letters requesting that information concerning calculators be sent to the home of an individual on the project staff were mailed to 39 firms identified as manufacturers of hand-held calculators. Returns were as follows:

3 - returned by post office

7 - replies indicated company does not make hand-held calculators

14 - sent advertisements for hand-held calculators

15 - no response (These included some now known to be distributors rather than manufacturers.)

Replies were received from the following companies:

Summit
Casio
Melcor
Unitrex
Craig
Toshiba
Rockwell
Radio Shack
Lloyds
Sharp
Monroe
Texas Instruments
Adler
Hewlett Packard

The advertisements received were examined to determine answers to the following questions:

(1) How many companies manufacture --

at least one full scientific model? 9

at least one programmable model? 1

at least one "four-function-only" model? 12

at least one four-function plus % model? 13

(2) Were prices supplied?

yes for all models: 4 yes for some models: 1 none: 9

(3) Was warranty information supplied?

yes for all models: 6 yes for some: 1 none: 7

(Warranties reported ranged from 90 days to 1 year. Some companies are consistent across models; for others length of warranty increases with price.)

(4) How many models were advertised?

Number of models	1	2	3	4	5	6	7	8	9	10
Number of companies	1	2	0	1	1	3	3	1	1	1

Data concerning features available on different machines were tabulated and are summarized in Table 1.

Notes on Advertisements (Table 1)

- Floating decimal: almost all models claimed to have this feature. Sometimes a mysterious "wraparound" decimal is mentioned. One model (TI 2550) has a floating/preset option.
- Other features: since this is advertising, some ambiguity exists. Some ads spoke of %, squaring, and other operations as "hidden features". If the pictured model did not show a % (resp x^2) key, these were not tallied. Similar ambiguity exists with regard to the "constant" feature and memory.
- Scientific notation figures are not necessarily accurate. (Use of this notation was tallied only when explicitly mentioned - presumably the 20 or so "full scientific machines" use it and others may as well.)
- The "repeat function" is mentioned by only one company.
- Not tallied: "true balance"; ability to parenthesize; number of memory registers; number of constants; functions for which constants are available.
- Rechargeable feature: some companies listed information about power sources while others did not. Only one listed the cost of separate adaptors. For some calculators, adaptors are included in the sale price.

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Table 1

Manufacturer number	1	2	3	4	5	6	7	8	9	10	11	12	13	14		Total
Number of models	8	1	10	9	7	2	7	4	6	6	2	7	6	5		80
Rechargeable?																
Adapter cost																
+, -, x, ÷	8	1	10	9	7	2	7	4	6	6	2	7	6	5		80
$\sqrt{}$	2	1	5	2	5	0	2	3	2	3	0	1	4	3		33
scientific notation	0	0	2	4	1	0	0	0	3	1	0	?	0	2		13+
floating decimal																
log (base 10)	0	0	1	3	3	0	1	1	4	2	0	5	1	1		22
ln (base e)	0	0	2	3	2	0	1	1	4	2	0	5	1	1		22
10^x	0	0	1	3	2	0	0	0	3	2	0	5	0	0		16
e^x	0	0	2	3	2	0	1	1	4	2	0	5	1	1		22
x^y	0	0	0	3	2	0	0	1	4	1	0	5	1	1		18
sin	0	0	2	2	2	0	1	1	4	2	0	5	1	1		21
cos	0	0	2	2	2	0	1	1	4	2	0	5	1	1		21
tan	0	0	2	2	2	0	1	1	4	2	0	5	1	1		21
arcsin/cos/tan	0	0	2	2	1	0	1	1	3	2	0	5	1	1		19
hyperbolic sin/cos/tan	0	0	0	2	0	0	0	0	1	0	0	0	0	0		3
n!	0	0	0	2	0	0	0	0	3	1	0	0	0	0		6
$x \leftrightarrows y$	0	0	1	2	0	0	0	1	4	3	0	7	2	1		21
Memory	4	0	5	4	2	1	4	2	6	4	0	7	4	3		47
Constant	5	1	?	6	7	2	3	4	4	6	2	7	5	4		66
+/- exchange	3	0	7	3	1	0	3	0	5	0	0	0	3	2		27
% key	3	1	7	5	4	0	4	2	1	2	1	2	3	3		38
π	1	0	6	1	2	0	2	1	4	1	0	5	1	1		25
1/x	3	0	4	4	1	0	2	1	5	3	0	5	2	3		33
x^2	2	0	2	2	0	0	0	0	5	1	0	0	1	1		14
Deg/Radian conv.	0	0	2	0	0	0	0	0	4	2	0	5	0	0		13
Metric conv.	1	0	0	0	0	0	0	0	0	1	0	0	0	0		2

Summary of Data from Advertisements

Overall there seem to be five basic types of hand-held calculators available:

1. Scientific
2. Four functions with one or two other keys (often %, x^2, 1/x)
3. Four functions only (with or without constant)
4. High-powered business or statistical models
5. Miscellaneous

Table 2
Types of Calculators Available from 14 Manufacturers

Manufacturer

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Type 1	1	0	2	3	2	0	1	1	4	2	0	5	1	1
2	5	1	6	5	4	1	6	2	2	3	1	0	4	4
3	0	0	0	1	0	0	0	0	0	0	0	1	0	0
4	2	0	2	0	1	1	0	1	0	1	1	1	1	0
5	0	0	0	1	0	0	0	0	0	0	0	0	0	0

B. Additional Data

Data on prices and features available on 15 models of hand-held calculators was reported by Budlong (1972). These data were reorganized to provide the price-by-features array provided in Table 3.

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Table 3

Company	Monroe	HP	Sharp	Monroe	HP	HP	Kingspoint	TI	Rockwell	Bowmar	Sears	Rem.	Sinclair	Commodore	Casio
Model Number	326	65	PC1002	324	45	35	SC40	SR50	202SR	MX100	ESR	SSRB		SR1400	FX10
Current Retail Cost	1200	795	645	500	325	225	170	150	120	120	120	100	100	100	100 ,
Rechargable?	✓	✓	ac.	✓	✓	✓	✓	✓	✓	✓	✓	✓	no	✓	✓
# of digits- mantissa	13	10	10	13	10	10	10	13	8	8	8	8	5	10	8
exponent	2	2	2	2	2	2	2	2	0	0	0	0	2	2	0
$\sqrt{}$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓
log (base 10)	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
ln (base e)	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓
10^x	✓	✓	✓	✓	✓								✓		
e^x	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓
x^y	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	1-9		✓	1-9
sin/cos/tan	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
arc sin/cos/tan	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	
sin h/cos h/tan h			✓					✓							
π	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓
$1/x$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓
x^2	✓	✓			✓		✓	✓			✓			✓	
Degree/radian conv.	✓	✓	✓	✓	✓			✓	✓		✓				
Degree/decimal conv.	✓	✓	✓	✓	✓							✓			✓
Polar/rectangular conv.	✓	✓	✓	✓	✓										
Metric conv. (# functions)		24			3										
Number of keys	43	35	29	43	35	35	38	40	20	25	25	29	18	36	29
Storage	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		✓	✓	
Parentheses (#)	4	4	0	2	4	4	2	0	0	0	0	0	2	2	0
Storage Registers (#)	12	9	8	10	NR	1	1	1	1	1	1	0	0	1	0
Programmable?	✓	✓	✓	✓											

Appendix B:

2. Summarized Responses from State Supervisors of Mathematics

The questionnaire (pages B-18 to B-19) was sent to 86 persons in the state departments of education and to 13 Canadian supervisors. Responses were received from 65 persons in 33 states and several provinces. In two instances, the state supervisor sent copies of the questionnaire to county or local educational agencies in the state, and these responses are collated separately (n=29).

Survey on Calculators　　　　　　Sample: Supervisors

1. Do you believe that hand-held calculators -- (check any which apply)

___ a. should be available for use in every mathematics class?

___ b. should be used if some students have them?

___ c. should only be used if all students have them?

___ d. should not be used in elementary schools (K-6)?

___ e. should not be used in secondary schools (7-12)?

___ f. should be used in the classroom but not on tests?

Comment in relation to your answer: ______________________________

____

____

____

____

____

2. Should hand-held calculators --

___ a. be provided by the school?

___ b. be provided by the child?

___ c. I don't believe they should be used in schools.

3. Estimate the use/non-use of hand-held calculators in your state:

number of schools using	________	number of students	________
number of schools banning	________	number of students	________
number of schools studying use	________	number of students	________
number of remaining schools	________	number of students	________

4. How do you believe that hand-held calculators should be used? (check any which apply)

___ a. checking answers (% of this use should be: ______)

___ b. routine calculation (% of this use should be: ______)

___ c. concept development (% of this use should be: ______)

___ d. open exploration (% of this use should be: ______)

___ e. ________________ (% of this use should be: ______)

___ f. ________________ (% of this use should be: ______)

___ g. ________________ (% of this use should be: ______)

___ h. ________________ (% of this use should be: ______)

5. Which use of hand-held calculators do you feel is most important at each of the following levels?

   a. at primary level (K-3) ____

   b. at intermediate level (4-6) ______________________________________

   c. at junior high level (7-9) ______________________________________

   d. at senior high level (10-12) _____________________________________

6. What modifications do you believe should be made in the K-12 curriculum if/when hand-held calculators are made readily available to students at all times? Star the three you believe to be most significant.

1. *Do you believe that hand-held calculators*

   a. should be available for use in every mathematics class? 52

   b. should be used if some students have them? 22

   c. should only be used if all students have them? 9

   d. should not be used in elementary schools (K-6)? 8

   e. should not be used in secondary schools (7-12)? 1

   f. should be used in the classroom but not on tests? 8

   no answer 2

Comments in relation to answers:

- Should not use calculators in classrooms unless all students have them.
- Could use calculators on tests where objective is not to test computational skills.
- If goal is computation with basic facts, calculator offers no advantage; when goal is to learn which operations to solve problems, discover alternate solutions, check solutions, etc., calculator then becomes very useful.
- Should be available from 3rd or 4th grade up after basic fundamental calculations are known.
- Not necessary for all students to have a calculator. Should be available like pencil sharpener - for use as needed.
- The use of hand-held calculators is both economically and educationally sound. The trend is for costs to be reduced even further. They will be the greatest asset to the teacher and student in the math classroom.
- Children need to develop a sense of number prior to their use.
- The use of calculators in elementary schools (K-6) would hinder the mastery of basic computational skills.
- Pupils should be allowed to use their own calculators.
- Calculators should be used any time that they would help teach a concept.
- Hope to see more and more hand calculators in use with intelligent supervision and encouragement.
- The sophistication of the calculator should be consistent with the ability of the students (e.g., no need of a square root capability for 2nd graders).
- There is a real issue with regard to the equality of educational opportunity where the use of hand-held calculators is concerned. If hand-held calculators are to be used in a classroom in a district, then all children in that district should have equal opportunity for access to them.

- No objection to their use on tests if all students have access to them and are capable of using them. Opportunities should be found for all students to have this experience.
- Would prefer to see the student have a desk-type calculator with a tape.
- In time, the hand calculator will become a common item in the classroom, especially in the senior high levels.
- There does not have to be a 1-1 correspondence between students and calculators. Calculators should be used on tests but all students do not need to be tested at the same time.
- Available in every classroom for every student but not necessarily to be used by all students. The appropriateness of using hand-held calculators has to be determined by each learner on the basis of his understanding and application of computational skills.
- The calculator should be used if the teacher is willing to do some individual work with students. Possibly through mini-courses students who have calculators could better learn their applications.
- Pebbles-in-the-sand, count-on-fingers, Napier's Bones, slide rule, logarithms - and now electronic calculators. All are helps for man's work in math. All have been used in classrooms. It's the future, that's all, friends.
- Use should be based upon the objectives of the course. There are times when they could be used in any course.
- Students need to know how to use a calculator properly.
- Calculators should not be used to replace normal paper-and-pencil calculations.
- If they are used by a class during instruction, they should be utilized on evaluation of instruction.
- Let the choice of using a calculator be determined by the learner.
- Should be available on a school or departmental basis but to put them in every classroom at this time is an unjustifiable expense.
- Calculators should be provided by the child because of maintenance; put 2 or 3 in each classroom for use by those who cannot afford their own.
- There are many variables that have to be identified before a 100% accurate answer is possible, e.g., has (in the elementary program) a material/conceptual approach been used as an introduction?
- The teacher should be prepared to build use of the calculator into the math program. They do not replace the students' needs for basic skills but can enhance the learning of them if there is planning in the way they are used.
- Preferred response is that hand-held calculators should be available in math classes for use with slow learners in attracting and sustaining interest; for special projects; and for verification of results or for rapid determination of results. At no time should it interfere with a student's learning computational processes or mastery of algorithms.

- The mini-calculator has arrived - students have them and if properly acknowledged in the classroom can be a great aid in understanding, motivation, and applications to problem solving. They pose real problems regarding curriculum reassessment. We can't pretend they don't exist, nor should we.
- Calculators can become an integral part of math programs; allow for extending the math to be taught; allow more balanced programs on all problem solving skills; and even fulfill a utilitarian role which is steadily increasing.

2. *Should hand-held calculators*

   a. be provided by the school? 56

   b. be provided by the child? 28

   c. I don't believe they should be used in schools. 0

   no answer 4

3. *Estimate the use/non-use of hand-held calculators in your state (rounded totals):*

   a. number of schools using: 4,800 *number of students* 500,000

   b. number of schools banning: 300 *number of students* 75,000

   c. number of schools studying use: 3,200 *number of students* 750,000

   d. number of reamining schools: 20,000 *number of students* 7,000,000

   e. not known or no answer: 40 states

4. *How do you believe that hand-held calculators should be used?*

	a. checking answers	*b. routine calculation*	*c. concept development*	*d. open exploration*
100%	7	1	1	1
80%	-	1	-	-
75%	1	-	-	-
65%	-	1	-	-
60%	2	2	-	-
50%	4	7	2	2
45%	-	1	-	-
40%	1	-	-	2
35%	2	-	2	-
30%	1	4	3	6
25%	2	-	2	4
20%	4	6	7	3
15%	1	2	1	1
10%	7	8	1	6
5%	3	-	2	3
2%	1	-	-	-
no % given	28	30-	23	28

other responses:

e) problem solving: 4 ; (30%)
f) motivation: 3 ; (50%)
g) tests: 1 ; (10%)
h) routine calculations in higher math: 1
i) non-routine calculations: 1 ; (20%)
j) algorithm development: 1 ; (20%)
k) fun activities: 1 ; (1%)
l) time consuming math, e.g., physics: 1
m) employment skills: 1 ; (10-15%)
n) business math: 1
o) student determined: 1 ; (100%)
p) "real life" problems: 2
q) math recreations: 1
r) number theory: 2
s) statistics: 1
t) use in science: 1
u) directed discovery: 1 ; (15%)
v) games: 1

5. Which use of hand-held calculators do you feel is most important at each of the following levels?

	primary (K-3)	intermediate (4-6)	junior h. (7-9)	senior h. (10-12)
motivation	2	4	2	3
checking answers	13	26	22	10
problem solving	-	2	6	9
explorations & discovery	16	20	18	19
alternate solutions	-	-	1	-
real life problems	-	-	-	1
long calculations	-	1	2	5
enrichment	-	1	1	-
concept development	9	16	12	5
routine calculations	3	10	24	27
science	-	-	1	4
operation order	3	2	-	-
estimation	-	1	2	1
algorithm development	-	1	-	-
decimal numeration	2	3	2	-
number patterns	1	2	2	1
employable skills	-	-	-	1
determined by student	1	1	1	1
full use	2	2	6	8
math electives	-	-	2	4
reinforce basic facts	1	-	-	-
properties of numbers	-	1	1	-
of no use	10	5	2	2
no answer	19	16	8	7

6. *What modifications do you believe should be made in the K-12 curriculum if/when hand-held calculators are made readily available to students at all times? Star the three you believe to be most significant.*

- More emphasis on problem solving. n=10
- Emphasize decimals and/or earlier introduction of decimals. n=18
- Emphasize estimation. n=11
- Less time spent on computation; more time on concepts. n=14
- More emphasis on approximation in measurement and study of round-off error. n=4
- Use of calculator for "real life" or practical, realistic problems. n=13
- Work with sequences, limits, maximums and minimums. n=3
- More emphasis on algorithmic process. n=2
- Integration with other disciplines (science) and more varied math courses (number theory, statistics). n=15
- Strengthening of fundamental skills. n=6
- Open exploration encouraged. n=5
- Flowcharting technique. n=3
- Don't modify on elementary and/or secondary level. n=6
- Use for checking. n=2

Other Respondees

1. Do you believe that hand-held calculators

a. should be available for use in every mathematics class? 9

b. should be used if some students have them? 4

c. should only be used if all students have them? 0

d. should not be used in elementary schools (K-6)? 18

e. should not be used in secondary schools (7-12)? 6

f. should be used in the classroom but not on tests? 7

no answer 1

2. Should hand-held calculators

a. be provided by the school? 11

b. be provided by the child? 7

c. I don't believe they should be used in schools. 9

no answer 0

3. Estimate the use/non-use of hand-held calculators in your state (rounded totals):

a. number of schools using: 98 number of students 2,625

b. number of schools banning: 2-3 number of students 2,500

c. number of schools studying use: 54 number of students 60,000

d. number of remaining schools: 176 number of students 89,244

e. not known or no answer 16

4. How do you believe that hand-held calculators should be used?

	a. checking answers	b. routine calculation	c. concept development	d. open exploration
100%	2	1	-	1
90%	1	1	-	-
85%	-	-	-	1
80%	2	-	-	-
65%	1	-	-	-
50%	2	-	-	2
30%	1	1	-	-

	a. checking answers	b. routine calculation	c. concept development	d. open exploration
25%	-	-	-	1
20%	-	1	-	1
15%	-	-	-	1
10%	-	-	2	-
5%	1	1	1	1
3%	-	-	-	1
no % given	10	7	5	7

other responses:

e) math labs: 10%

5. *Which use of hand-held calculators do you feel is most important at each of the following levels?*

	primary (K-3)	intermediate (4-6)	junior h. (7-9)	senior h. (10-12)
checking	2	9	14	9
exploration & discovery	4	3	4	8
concept development	3	3	4	5
routine calculations	-	4	6	11
full use	-	-	-	1
math electives	-	-	-	3
of no use	8	6	2	-
no answer	10	8	5	3

6. *What modifications do you believe should be made in the K-12 curriculum if/when hand-held calculators are made readily available to students at all times? Star the three you believe to be most significant.*

- More emphasis on problem solving. n=3
- Employing estimation. n=1
- Less time on computation; more time on concepts. n=2
- Work with sequences, series, limits. n=1
- Integration with other disciplines and math courses. n=2
- Strengthen fundamental skills. n=2
- Open exploration encouraged. n=2
- Use for checking. n=2
- Don't modify. n=7

Appendix B:

3. Summarized Responses from School Personnel

The questionnaire (pages B-28 to B-30) was sent to 32 teachers and 26 other school personnel; responses were received from 16 teachers and 16 others, in 20 states.

Survey on Calculators Sample: school personnel

1. Do any pupils in your school(s) use hand-held calculators in classes?

   Yes _____ If yes, please answer questions 2-11.

   No _____ If no, please answer questions 12-15.

2. Who provides the hand-held calculators?

   ___ a. Pupils/parents (approximate number: ______)

   ___ b. School/school board

   Manufacturer: ____________________ Model ____________________

   Number purchased: ____________ When ? ____________________

   Are you pleased with this selection? Yes ___ No ___

   Why or why not? ____

   ____

   ____

   [Use reverse side of this page if more than one type purchase

   How was the decision to provide hand-held calculators reached?

   ____

   ____

   ____

   ___ c. Other suppliers -- please specify: ______________________

   ____

3. In what subject areas are hand-held calculators used? (check any which apply)

   ___ a. Mathematics

   ___ b. Science

   ___ c. Business education

   ___ d. Other -- please specify: ______________________________

4. Are hand-held calculators used -- (check one)

   ___ a. only in classes?

   ___ b. only on tests?

   ___ c. both in classes and on tests?

5. Is the policy on the use of hand-held calculators determined by -- (check one)

   ___ a. the individual?

   ___ b. the faculty as a group?

   ___ c. the principal or other administrator/supervisor?

   ___ d. the school board?

6. How many students are in your school(s)? __________

7. What suggestions do you have for those who are selecting a hand-held calculator for school use?

____

____

____

____

____

8. What security measures are used to protect school-owned hand-held calculators?

____

____

____

____

9. What is the attitude of the majority of parents (public) in your district about the use of hand-held calculators in the schools?

___ a. favorable

___ b. unfavorable Why? ______________________________

___ c. controversy, but no clear majority

___ d. no reaction

10. How do you believe that hand-held calculators should be used? (check any that apply)

___ a. checking answers (% of use should be: ______)

___ b. routine calculation (% of use should be: ______)

___ c. concept development (% of use should be: ______)

___ d. open exploration (% of use should be: ______)

___ e. ________________ (% of use should be: ______)

___ f. ________________ (% of use should be: ______)

___ g. ____________ ____ (% of use should be: ______)

___ h. ________________ (% of use should be: ______)

11. Which use of hand-held calculators do you feel is most important at each of the following levels?

a. at primary level (K-3) ______________________________

b. at intermediate level (4-6) ______________________________

c. at junior high level (7-9) ______________________________

d. at senior high level (10-12) ______________________________

12. Why are hand-held calculators not used in your school(s)? (check any which apply)

___ a. Their use is banned or prohibited. By whom? ____________________

___ b. No child or teacher has expressed interest in using hand-held calculators in school.

___ c. The question has not been raised.

___ d. Other -- please specify: ____________________

____

13. How many students are in your school(s)? __________

14. What do you feel are the best reasons for not using hand-held calculators in schools?

____

____

____

____

____

____

____

____

____

____

15. Do you foresee any problem(s) if students used hand-held calculators for doing homework (out of school)?

____

____

____

____

1. *Do any pupils in your school(s) use hand-held calculators in classes?*

   Yes: 22 *(Answered only questions 2-11.)*

   No: 10 *(Answered only questions 12-15.)*

2. *Who provides the hand-held calculators?*

   a. *Pupils/parents:* 10

   b. *School/school board:* 17

				Pleased	
Manufacturer	*Model*	*# purchased*	*When*	*Yes*	*No*
Texas Instruments	--	11	Spring 1974	x	
	SR-50	2	—	-	-
	2500	30	—	-	-
	2500-II	30	March 1975	x	
	SR-10	16	1974		x
	1500	1	—	-	-
Casio	8M	16	1974	x	
	—	240	May 1975	x	
	personal mini	1	June 1975	x	
Bomar	MX-50	30	Sept. 1973	x	
Corvus	401	30	Feb. 1975	x	
	312	30	Oct. 1974	x	
Adler	805 NAD & 815 NAD	198	April 1975	too soon to tell	
Sharp	ELSI 8101	35	1974 & 1975	x	
	ELSI-8; 801; EL 122; 8002	8	1972 & 1974	x	
RES	Mach-II	19	Oct. 1974		x

Why or why not pleased? How was the decision to provide hand-held calculators reached?

- TI 2500-II; replaceable battery proved to be better than rechargeable battery where battery could not be replaced without soldering; decision to use as part of NSF problem solving project

- TI SR-10; not pleased because prefer TI 11 or HP 21; no information on decision.

- CASIO 8M; have been trouble free to date; decision reached by teachers and administrators.
- CASIO; seems durable; decision reached through acceptance of a proposal for use in two districts.
- CASIO-personal mini; works well except doesn't have adaptor; decision reached by 6th grade students who decided to spend class money for one.

- BOMAR MX-50; seems sturdy, minimum of malfunctions, four calculation processes available on all machines, handled all math necessary for Grade 6; decision reached from pilot program by State Education Department.

- CORVUS Mode 312; have had good service during one year's use; decision in relationship with a Math-Lab set up with Title II and III funds.
- CORVUS 401; service free, serves instructional needs of a K-9 program; decision reached through proposed research project and cooperation of participating schools.

- ADLER 805 NAD and 815 NAD; too soon for evaluation; decision by Secondary Math Curriculum Committee made up of department chairmen for grades 7-12.

- SHARP ELSI 3101; has good capability and have had excellent service; decision reached by department heads
- SHARP ELSI 8, ELSI 801, EL 122, EL 8002; dependable, reasonably inexpensive, have very plain display panels, good size keys, simple to operate, and well built; decision by Title III minigrant to math lab school and by principal.
- RES Mach II; don't stand up.

c. *Other suppliers:* Commodore; Accutron; Ardvark; HP 21, 35, 45; Disi-matic; Rapidman 1212; Rapid Data Systems.

3. *In what subject areas are hand-held calculators used?*
   a. *Mathematics:* 22

   b. *Science:* 10

   c. *Business education:* 5

   d. *Other:* 3

4. *Are hand-held calculators used --*

   a. *only in classes?* 14

   b. *only on tests?* 1

   c. *both in classes and on tests?* 10

5. *Is the policy on the use of hand-held calculators determined by --*

   a. *the individual?* 11

   b. *the faculty as a group?* 8

   c. *the principal or other administrator/supervisor?* 2

   d. *the school board?* 1

6. *How many students are in your school(s)?*

   Total number of students in schools: 971,781

7. *What suggestions do you have for those who are selecting a hand-held calculator for school use?*

   - Batteries should be type that can easily be replaced. Heavy use was too much for non-replaceable, rechargeable batteries.
   - Teachers and students prefer larger display of Monroe over Texas Instruments.
   - Percent-key of little value, if any.
   - Logic of Texas Instruments is best; Monroe does not have appropriate logic, however students (4-6) adjusted easily and did not complain.
   - Get all teachers involved in the selection.
   - (a) rechargeable or not (non-rechargeable seems best)
     (b) type of batteries used
     (c) crystal display
     (d) trade-in value after one month
     (e) can dust get in machine
     (f) are batteries separate from rest of machine
     (g) number of digits in read-out
   - Make clear agreements with suppliers about replacements. A system should be set up in advance so that when breakdowns occur, the company will immediately supply a replacement.
   - Overflow indicator.
   - Clear entry key.
   - Quality, desirable case (especially for grades 3-8).
   - Functions for grades 9-12: a^x, trig and inverse functions, exponential notation, M^+, M^-, indicator for memory, parenthesis.
   - Operable on both AC and DC.

8. *What security measures are used to protect school-owned, hand-held calculators?*

- Each teacher responsible for his own calculator.
- Make sure if loaned out, it is checked in each period.
- Some are secured, while most are on a check-out system where they all have to be turned in before any class or child leaves.
- Locked cabinets with recharging plugs inside. Preferably in a room where units are used as opposed to a hallway facility (closet).
- All calculators have a school number and name engraved on the back. Calculators are left in boxes of 15. Calculators are checked out in numbers of 15 or 30. One teacher is in charge of locking them up each evening.
- Attach to a board; if possible, pair so students sitting opposite each other could use them.
- Assign calculators to individuals.

9. *What is the attitude of the majority of parents (public) in your district about the use of hand-held calculators in the schools?*

a. favorable: 9

b. unfavorable: 1 *Why?* Community is still pretty traditional.

c. controversy, but no clear majority: 2

d. no reaction: 8

10. *How do you believe that hand-held calculators should be used?*

	a. checking answers	*b. routine calculation*	*c. concept development*	*d. open exploration*
100%	2	1	1	2
80%	-	1	-	-
60%	1	-	1	-
50%	1	1	1	-
45%	1	-	-	-
40%	-	2	1	-
30%	1	1	-	2
25%	-	-	3	1
20%	3	1	1	2
15%	1	2	1	1
10%	3	1	3	4
5%	-	1	-	-
no % given	7	7	7	8

other responses:

- *e) problem solving* (10%, 20%, 35%, 100%)
- *f) integrated curriculum* (25%)
- *g) estimation* (100%)
- *h) use in place of tables* (20%)
- *i) remedial work*
- *j) consumer topics*
- *k) career math*
- *l) research*
- *m) games* (5%)

11. Which use of hand-held calculators do you feel is most important at each of the following levels?

	primary (K-3)	*intermediate (4-6)*	*junior high (7-9)*	*senior high (10-12)*
checking answers	5	9	6	5
routine calculations	-	4	8	9
concept development	6	7	7	7
open exploration	9	9	6	7
problem solving	-	3	1	2
widen experiences	-	1	1	-
long calculations	-	-	-	1
number development	1	-	-	-
estimation	-	-	1	1
remedial work	-	-	1	-
research	-	-	1	1
full use	2	2	2	3
little or no use	3	-	-	-
no ans. or opinion	2	1	3	2

12. Why are hand-held calculators not used in your school(s)?

a. Their use is banned or prohibited. 0 *By whom?* ____________

b. No child or teacher has expressed interest in using hand-held calculators in school. 1

c. The question has not been raised. 4

d. Other. 8

- There have been some ordered for future use.
- Lack of funds to provide calculators to students.
- A few will be provided for the 1975-76 school year.
- Only certain individuals have expressed the desire to use them and they have been purchased by these individuals.
- We prefer desk models which can be locked in place.
- We do not want the problems associated with batteries and recharging units.

- Teachers have discussed the possibility but no requests have been made.

13. *How many students are in your school(s)?*

Combined number of students in all schools responding: 225,350.

14. *What do you feel are the best reasons for not using hand-held calculators in schools?*

- Perhaps availability of hand-held calculators would inhibit student learning of tables of number facts. Perhaps it would militate against students developing ability to do arithmetic calculations or estimations mentally. Perhaps it would make students too dependent on a mechanical computing aid. BUT on balance, I see the advantages of hand-held calculators outweighing the disadvantages.
- If students fail to learn basic math when they are capable of achieving mastery without the calculator, use of the latter should be restricted. Total dependence on the calculator should not occur.
- We do not want the problems associated with batteries and recharging units. We do use small desk calculators.
- Cost.
- Some students would tend to depend on calculators instead of learning basic facts in mathematics.
- Who will be able to use them and when? It would be difficult to provide everyone with a calculator when he needs it (or wants it). Too expensive for the school system to purchase.
- None. I feel hand calculators would definitely be an asset in class.
- None. I think it's about time we began and I think they will soon be used.
- I have none. We have been operating on a reduced budget and could not afford them.
- Too costly to provide one for each student.

15. *Do you foresee any problem(s) if students used hand-held calculators for doing homework (out of school)?*

- No. Supply of calculators in an elementary school presents problems: initial cost, possibility of theft, difficulty of getting all students to supply their own, and not least important - the P.R. problem of convincing parents that their children will learn how to compute more effectively if they have calculators than if they do not.

- No, unless the calculator is not accurate. Students who cannot achieve mastery should be permitted to use calculators both in school and at home.
- The response would be dependent upon the level: computational skills-yes; advanced mathematics-no.
- Depends on purpose of work.
- No!
- Possibility of not learning the processes and it certainly will be necessary to do "manual" calculating at times so it must be learned. Danger of loss of school calculators if they are taken home.
- Students would possibly be unaware where or why they had mistakes.
- Yes, in the area in which I teach, however, they might disappear!
- No!
- No. I think it would free the student of long calculations and provide them more time to concentrate on theory.

Appendix B:

4. Summarized Responses from Teacher Educators

The questionnaire (pages B-39 to B-40) was sent to 87 teacher educators; responses were received from 78, from 39 states. Responses to questions 1 and 2 are not included, since these questions were asked only to generate names (the school personnel questionnaire was sent to all those named).

Survey on Calculators

Sample: teacher educators/developers

1. Are you aware of schools or teachers who are banning the use of hand-held calculators in the classroom or during tests?

Name	Location	Contact person	Banning		
			everywhere	classes	tests

2. Are you aware of schools or teachers who are enthusiastically endorsing the use of hand-held calculators in the classroom?

Name	Location	Contact person	Type of use

3. What do you consider the three most compelling arguments for the use of hand-held calculators in the classroom?

4. What do you consider the three most compelling arguments for the banning of hand-held calculators in the classroom?

5. As an expert in mathematics education, what recommendations would you give to elementary and secondary schools regarding the use of hand-held calculators today?

6. What modifications do you believe should be made in the K-12 curriculum if/when hand-held calculators are made readily available to students at all times? Star the three you believe to be most significant.

7. Assume that a school system (K-12) has moved to widespread use of hand-held calculators throughout the curriculum and testing program. React to each of the following in such a framework:

True	False	
___	___	a. The mathematics curriculum would need to be modified extensively.
___	___	b. The mathematics learned in elementary schools would be significantly less.
___	___	c. Students would no longer remember the basic facts.
___	___	d. Parents would strongly oppose such widespread use of hand-held calculators.
___	___	e. Elementary teachers would not use hand-held calculators during mathematics lessons.
___	___	f. Students would no longer have an appreciation for concepts such as negative integers, fractions, square roots, etc.
___	___	g. Fractions will no longer be taught in the elementary school.
___	___	h. Students would improve in the ability to estimate quantities.
___	___	i. Students would improve in the ability to solve problems.
___	___	j. Students would lose interest in mathematics.
___	___	k. Students would become lazier than they are.
___	___	l. Students would lose confidence in their ability to handle numbers.
___	___	m. Students would have a greater understanding of concepts such as square root, negative integers, fractions, etc.
___	___	n. Students would gain understanding of the decimal system.
___	___	o. Students would gain interest in arithmetic.

3. *What do you consider the three most compelling arguments for the* *use* *of hand-held calculators in the classroom?*

- More attention can be given to problem solving and to estimating answers.
- More attention can be given to geometry, probability, graphing and number theory.
- xxx

- They are here to stay and are part of one's everyday experience.
- Their practical use in computation.
- Their use as a motivational factor.

- "Flexible Answer Key" concept.
- Resource tool which promotes student independence.
- Encourages inquisitive and creative experimentation.

- If properly used, they can facilitate concept development.
- An asset in teaching problem solving.
- A much more efficient tool than slide rules and should make the slide rule extinct.

- Saves time in tedious calculation.
- It may well free the student to explore in areas in which calculation might discourage him.
- xxx

- They serve as an aid to calculation allowing more realistic mathematical examples to be used.
- They eliminate the need for as much time spent in rote memorization of facts and skill development.
- They are here. To ban them is comparable to insisting on memorization of facts in a reference book.

- To compute "partial sums, products . . ." in learning algorithms.
- Reinforcement in basic fact drill activity.
- Reduce computational "load" in problem solving.

- They are available; we must cope with their existence.
- Reinforcement after "student" computation.
- Use in problem solving to minimize computation aspect.

- They give correct answers.
- They are and will be available to educated people.
- They allow us to do more and know more than otherwise.

- Motivation.
- Problem and answers not otherwise possible.
- Time, efficiency, accuracy.

- Their availability.
- Their usefulness as an aid in solving problems.
- Their use in generating mathematics thinking, i.e. patterns, functions, operations, etc.

- It removes some of the "pressures" of computational problems during problem solving.
- It might generate some interest in mathematics and improve attitude.
- It might make the classroom situation more realistic to some students.

- Save time. Why do all of the work by hand when a machine can do it faster.
- In the near future many of us will be using calculators daily so now is the time to teach their use.
- Convenience.

- No real need to be able to perform tedious calculations, but should know how to perform them.
- Hand calculators enable students to explore problems and situations not accessible otherwise due to complicated or extensive calculations needed.
- Very motivating and a very real part of our current technological society.

- They are now readily available and cheap.
- Some studies have indicated improved learning of mathematics and arithmetic skills.
- More and better mathematics could be taught with use of calculators.

- They permit independent investigation of many problems more easily.
- They eliminate teaching of calculation so that more attention can be given to the ideas involved.
- They permit problems to be more interesting because the arithmetic need not interfere.

- Speed and accuracy.
- They make it easy to test formula, e.g. to get a feeling for the variance of skewed distributions.
- xxx

- They teach the capability of calculators.
- Develop a positive attitude.
- Can be used to teach flow charting and related logic.

- They can enable students to handle larger and more complex problems.
- They are widely used in society and readily available.
- The remedial student who is poor in arithmetic can go ahead and experience success in mathematics.

- I view this as a function of level--argument for lower grades (e.g. develop basic addition and/or multiplication facts), may not be appropriate later.
- One that would cross all levels would be allowing students to concentrate on problem rather than on skills needed to solve the problem.
- xxx

- Free student time to concentrate more on mathematics, less on arithmetic.
- Redirect student attention to more significant aspects of problem.
- Speed and accuracy.

- They are becoming a reality of life; persons will have them, so should know how to use them.
- Permits student to look at structure of mathematical operations without arithmetic getting in the way.
- Motivational factor.

- They help--phony not to have them.
- Enable one to consider more kinds of problems, data, situations.
- No one has the percentage of accuracy that a calculator has.
- May aid understanding of math ideas if used properly.

- Calculators will be commonly available in 10 years or less.
- Speed and accuracy of solutions.
- Good motivational device.

- Their use might help to shift the elementary curriculum from an almost total preoccupation with how to compute to spending more time on when and why one computes.
- xxx
- xxx

- Reference.
- Opportunity to assign more complex calculations in problem solving, etc.
- Positive affective changes in kids.

- Can solve more realistic problems.
- Saves time from arduous computation.
- Student can concentrate on aspects of problem solving.

- You can do more sophisticated, realistic mathematics without being limited by substantial arithmetic.
- Encourages estimation.
- xxx

- Ease of computation.
- xxx
- xxx

- Checking answers.
- Enrichment.
- Develop interest in and encourage use of mathematics.

- Make calculations easy. (The only argument.)
- xxx
- xxx

- Permits students to focus on the content of problems rather than the calculations involved.
- Calculators are motivating for children.
- Calculators are (will be) a part of adults' experiences.

- Their use may lead to improvement in math skills and concepts.
- Their general use in society, i.e. engineering, business, etc.
- Their ability to do rapid calculations which decrease errors and permit additional work.

- Enlarge the number and kind of problems which can be solved and to help students keep their minds focused on their problem-solving techniques.
- Aid in the development of concepts.
- Aid in graphing "interesting" functions.

- Experience with them can be a lead-in to computer work.
- There are a couple of high school courses, statistics and calculus, where there is enough computation that they could be useful.
- One could utilize them to teach better organization of computation.

- Part of today's world.
- Helps child's understanding.
- Places emphasis on estimation.

- Motivation.
- A useful skill in itself; helps students add long sums, as in business and accounting.
- Illustrates the usefulness of numbers, etc.

- In the "real world" virtually all arithmetic is done by calculators.
- Emphasizes problem solving and planning rather than manual computation.
- Extends the range of computations which are "reasonable" in given time.

- Efficient and rapid for "mechanical" aspects of math.
- Will use in later life ("social utility").
- To learn other math concepts with time saved.

- They are available and can't be avoided.
- They can be a motivational element.
- They can serve to reinforce concepts.

- It is an efficient way to compute.
- They will be used throughout life.
- xxx

- Motivational for the slow and average who experience frustration in computation.
- Students will learn some of the number facts by constant repetition on the calculator.
- Immediate check on student paper and pencil computation instead of waiting for the teacher.

- Relieves child of frustration when unable to compute accurately.
- Allows the slow child to go on while working on these skills.
- xxx

- To free us from laborious computation.
- xxx
- xxx

- When appropriate, it allows student to focus on principles (a la Gagne) without great worry over computation and without undue fatigue.
- xxx
- xxx

- Easy time doing simple calculations so that more material or richer material can be developed.
- Provide reinforcement of basic facts and algorithms.
- xxx

- Can solve problems without confusion/mistakes during computation.
- Relief from tedious computations.
- Improvement in accuracy.

- Less time would be spent on paper and pencil drill.
- Greater speed and accuracy in computation should result.
- More time and a means would be available to develop greater understanding of concepts.

- Emphasizes conceptual development.
- Technology will eliminate need to memorize basic facts.
- Speed of processing and volume of work will demand it.

- Takes the "worry" out of arithmetic.
- xxx
- xxx

- Use of realistic problems.
- Emphasis on algorithms.
- Freedom from computation in problem solving.

- They eliminate the tedium of computation in situations in which the problem analysis is more important than the computation.
- Provide immediate feedback.
- Use represents reality in commercial and industrial situations.

- They are here, students have them, it is silly to say we can't use them.
- They are a useful tool; it is up to us to find ways to use them.
- They open new doors to problem solving.

- Saving in time, i.e. will tackle much more involved problems with larger numbers.
- Student is more apt to get overall perspective on problem; can concentrate on operations.
- xxx

- Motivation.
- Problem solving not inhibited by tedious computation.
- Understanding of use and limitations of algorithms process..

- As with most machines, it relieves the user of the drudgery of work (computation) to concentrate on the aspects so man's mind can be best used.
- It is logical that we should use the most effective means of teaching whatever it is we are teaching. The ultimate aim is to teach problem solving skills. Let's get with it and use the feasible technology that is available at reasonable cost.
- Telling students they cannot use a calculator is like telling a teacher that movie projectors, overheads, tapes, TV, etc. are not appropriate aids for teaching, or telling a doctor that the myriad of technology available is not to be used.

- Student can learn to use arithmetic; helpful to those who do not know their number facts.
- Motivation--students enjoy using calculators.
- Learning to operate calculators.

- Reduce the need for hours of drill on "hard" math operations such as 3d x 3d multiplication and long division.
- Speed computation in high school classes, such as trigonometry and analysis.
- xxx

- Realistic problems without tedious computation.
- Recognition of algorithmic procedures.
- Facility of considering greater variety of materials.

- Attitude.
- Understanding of algorithms.
- Problem solving capabilities.

- Student interest.
- More and a wider range of problems could be undertaken in the senior high school courses.
- The hand-held computer is becoming widely used.

- By-pass awkward computation to do true analysis and problem solving.
- To do "on the spot" exploration of mathematical ideas.
- To do real-life problems with realistic numbers.

- Students can concentrate on non-computational concepts.
- More advanced examples can be used.
- Students enjoy using them.

- They are available (kids will all have them).
- Aid to problem solving.
- xxx

- Allows more "real" problem work.
- Makes discovery and verification more easy and/or possible.
- Illustrates and reinforces concepts of approximation, error, round-off.

- Until we have some data on the effects of calculators, any opinion is baseless. So I have no opinions, suggestions, or recommendations.
- xxx
- xxx

- Problem solving where algorithmic skills are not the learning objective.
- Motivation.
- xxx

- Motivation.
- Makes mathematics more accessible as a tool.
- Can aid concept learning.

- Can be an aid and an incentive for learning more mathematics.
- Low cost and availability, along with usefulness.
- An aid in solving significant problems that couldn't be dealt with otherwise.

- Training in their use.
- Increased range of problem-solving situations.
- Motivation for computation practice.

- Less time would be spent on paper and pencil drill.
- Greater speed and accuracy in computation should result.
- More time and a means would be available to develop greater understanding of concepts.

- Highly practical.
- Leads to essentially error-free computation.
- Avoids much frustration among slow learners.

- Potential for exploring algorithmic processes.
- Fast computation.
- Examining simple arithmetic relationships.

- To get computations for solving problems of greater depth
- As part of future culture.

4. *What do you consider the three most compelling arguments for the banning of hand-held calculators in the classroom?*

- I can think of none.
- The usual reason given is that students would not learn the basic facts. (I do not go along with this reason.)
- xxx

- None--if used judiciously and not as a replacement for skill development.
- xxx
- xxx

- Lack of mastery of basic facts.
- xxx
- xxx

- Error prone.
- xxx
- xxx

- One should be able to do common algorithms with skill and speed.
- xxx
- xxx

- I don't consider any to be compelling. The major argument I hear is that kids still need to have the skills in arithmetic computation. Mainly, I think this is based on a fear by teachers that their old ways of testing kids will no longer work.
- xxx
- xxx

- When used as a substitute for learning arithmetic processes.
- xxx
- xxx

- Could become a "machine crutch" before concept mastery is established (dehumanizing).
- Unfair to classes of students from low economic area districts.
- Expensive, in general, if implemented on an educationally reasonable basis.

- Not everyone can buy one so it is unfair to allow anyone to use them.
- xxx
- xxx

- Students won't learn basic facts, especially if introduced too soon.
- Not all students can afford one.
- xxx

- The possible misuses that could be made of them . . . teaching how to use a calculator in lieu of teaching mathematics.
- xxx
- xxx

- It might diminish the basic skills for students when not handling the calculator.
- It might cause a dependence upon the calculator.
- It might put undue pressure on parents to purchase calculators for their children.

- Overuse. If student hasn't achieved competency in addition, subtraction, etc. without machine, he may never do so.
- Because of cost each student won't have access to a calculator . . . all should be treated equally.
- Cost . . . it's not fair that some have calculators and others do not.

- Students will use calculators as a crutch and won't learn how to compute.
- Just another fad that will pass.
- Too expensive for the benefit derived from their use.

- Unprepared teachers could teach the wrong things, e.g. compulsive skill emphasis.
- Need for more of a research base.
- Prospect of investing in very cheap machines which would not last.

- In lower grades students may fail to master the basic facts.
- Students may not master the four basic algorithms.
- Encourage mental laziness.

- Cost.
- Availability--the calculators differ greatly. I have seen some which do not even accumulate products!
- xxx

- Some students cannot afford them and in some test situations they are placed at a disadvantage.
- xxx
- xxx

- I have no arguments for banning their use. There may be times when their use is banned, i.e. to develop mental arithmetic skill.
- xxx
- xxx

- Lack of availability.
- xxx
- xxx

- Children won't be able to compute accurately.
- Students will depend too much on calculators.
- xxx

- Some students cannot afford them nor can the schools afford to supply them.
- Some students with physical problems will have difficulty.
- It is questionable what the benefits will be (testing).

- Students might use them when you wish to check their computational abilities.
- xxx
- xxx

- They are pretty useless except in a very few courses.
- A few students might not learn to compute.
- As a distracting toy, they might interfere with learning.

- Cost.
- Not integrated into program yet.
- Not yet understood.

- None, except when students are being tested on the ability to calculate.
- xxx
- xxx

- Child can't always have it with him.
- If one can't compute, one won't know when it's malfunctioning.
- xxx

- Misused by teacher and child (prior to understanding of concepts and computations).
- More available to those who can afford calculators (inequality).
- xxx

- Teachers will avoid teaching computational processes with understanding.
- Expense (and possibly rip-offs).
- Psychological impact of the calculator as a crutch.

- Children tend to not learn computations and algorithms.
- Expense.
- xxx

- Price. Some teachers feel that either all or none should have the calculator.
- Calculator does not meet the needs of all mathematics in the classroom; for example, computation with fractions. Students may forget the classroom algorithms for the various computations.
- xxx

- Students will become dependent on them; will become lazy.
- Will not develop the same computational skills.
- All students may not be able to afford one.

- Student becomes over reliant.
- The value is unknown.
- xxx

- Computational skills will decrease (don't feel this is valid).
- Knowledge of basic facts will fade (might be true).
- xxx

- Students may not learn their number facts and algorithms.
- Students may learn the buttons to push but not understand the process.
- xxx

- None, other than on tests in elementary.
- xxx
- xxx

- All students must have equal access to calculators.
- Rapidly changing technology with resulting changes in algorithms.
- Undue dependence of child upon them.

- Child becomes machine dependent.
- xxx
- xxx

- Might decrease students' ability to compute mentally or with paper and pencil, although the computer could be used in such a way as to enhance their computational skills.
- xxx
- xxx

- Misuse by teachers who do not have adequate math backgrounds to determine where or where not or how to use them.
- xxx
- xxx

- Meaning for operations may be lost.
- Skills may not develop properly for future needs.
- Teachers may not know how to incorporate them properly into the curriculum.

- Cost.
- xxx
- xxx

- If you mean outright prohibition, I do not see any compelling reasons. The use of calculators must be kept in perspective, however, as it can never completely replace consideration of 4 operations and manual calculations therein.
- xxx
- xxx

- Might be misused by some teachers and/or children.
- If not provided by a school district, it puts certain economic groups at a disadvantage.
- Presence of the calculators might cause an undue preoccupation with computational mathematics.

- There are no good arguments for banning totally. However, arguments against their constant use include: (1) hand-held calculators will never be omnipresent and (2) if all don't have them, some should not be able to use them on tests.
- xxx
- xxx

- May discourage learning of some primary facts.
- Unfair advantage if all students are not provided calculators.
- Cost.

- Lack of understanding of basic operations.
- Unfair advantage unless accessible to all students.
- xxx

- Used when what is wanted is paper and pencil work.
- xxx
- xxx

- Might not learn basic facts.
- Might become lazy and use calculator for simple problems that he could easily do himself.
- Expense of supplying each child with one.

- Over-use sometimes discourages thinking and skills.
- xxx
- xxx

- All students cannot afford an equivalent machine.
- xxx
- xxx

- De-emphasis of usual computation may take place.
- Children may gain false impression of what mathematics is.
- Structure and problem-solving may be de-emphasized.

- Cost to parents.
- For tests designed to test computational skills, using a calculator should be discouraged.
- xxx

- Relaxation of memorization of basic facts and essential algorithmic knowledge.
- xxx
- xxx

- Children use as a crutch and therefore don't learn number facts.
- Operate calculators with no math understanding.
- Cost.

- Maintenance problems related to the hardware and batteries.
- Children and adults may become somewhat dependent for calculations.
- May make teachers uneasy.
- None [of the above] are really compelling.

- Not available to all students, thus unfair competition or financial burden.
- Could result in crutch for those not learning basic arithmetic facts.
- Gives emphasis to product rather than mathematical process.

- In early years, can reduce practice and motivation for learning facts and algorithms which are still needed.
- Encourages exaggerated expectations from machines.
- Equates mathematics to computation.

- Replacement of algorithmic skill understanding and practice.
- xxx
- xxx

- If professional planning for effective use is not provided.
- xxx
- xxx

- The possibility that certain skills will not be reinforced sufficiently to be usable.
- xxx
- xxx

- Leads to dependence on them, i.e. basic skills will not be learned.
- Arithmetic may become a "black-box" mystery.

- May interfere with learning basic facts.
- May interfere with learning standard algorithms.
- May interfere with learning much about fractions.

- Be sure student does possess some skill in computation without calculator.

"None" (with no qualifications) was the response of 9 persons.

5. *As an expert in mathematics education, what recommendations would you give to elementary and secondary schools regarding the use of hand-held calculators today?*

- Spend much more time on multiplying and dividing by powers of ten and rounding numbers before obtaining an answer.
- Have contests to see how close students can come to answers by rounding and then using knowledge of powers of ten. Estimation would be checked by using the calculator.
- I think for the time being calculators should become a part of classroom instruction like pencils, paper and chalkboard in the intermediate grades. However, I would not advocate their deliberate use in the primary grades nor would I ban their use. I would encourage teachers to use the calculator as a way of introducing students to decimals, but I would not support efforts to use the calculator unless the goal of instruction is understanding and problem solving in the broadest sense.

- Learn how to use them yourself first.
- Develop units of work that incorporate them into regular class work.
- Search literature and obtain help from professional journals (MT and AT).
- Use for specific skills, i.e. estimation, concept development, checking, discovery, etc.

- Encourage teachers to experiment and not wait "for the word."

- Don't buy them until you know what to do with them.

- I can encourage use after some skill is developed in calculator without the calculator.
- Especially at the secondary level, use of calculators should be encouraged; also roots, involved multiplications, divisions, etc.
- I would encourage their use but would still want them to have skill in "doing them by hand."

- Change teaching emphasis to concept development and when to apply the various operations.
- Spend time on estimating reasonable answers to lessen the danger of mispunching to get wild result.
- Do not worry about developing speed of calculation (by hand) in elementary school kids.

- Use the type of calculator so that keypunchs, order, etc., corresponds to the order that the symbols are written.
- Use a type of calculator on which certain controls are possible so that the calculator can be restricted to certain operations, e.g. when kids do multiplication--2 digit x 2 digit--restrict the calculator to 2 digit x 1 digit multiplication so that the partial products need to be computed and recorded.

- Do not stick your head in the sand. They will not go away.
- Learn how to use them in an educationally sound manner.
- Do not be gullible to the strong commercial pressures from the electronics industry.

- Buy an algebraic language one and use it.
- Recognize the fact of their wide acceptance and availability to students, parents, everyone!
- Review your goals--are you teaching kids who will be in the real world (where hand-held calculators abound)?
- Study, read, devise ways to incorporate and capitalize on hand-held calculators in your class.

- Elementary school: have a few available for special situations. Do not introduce generally.
- Secondary: if they are to be used, must be available to all. Emphasize use in general math but limit use in college prep classes.

- Use them to aid in problem solving--to save time doing computations that students know how to do. Use them to explore relationships among operations, functions, etc.
- I would encourage a school system (math department) to plan how the use of calculators could be effectively accomplished, how they fit into the objectives of each course, and what new ways they could be used to motivate and stimulate creativity and interest in the students.

- None, until more research is done on the effects of the calculator upon mathematics teaching.

- The calculator, like other motivating devices, helps to motivate students. Use them for motivation, but don't over do it. Cuisenaire blocks are excellent for motivation but I don't want my child to always use them or instantly use them. Strike a happy medium.

- Put them in the classrooms and watch what happens. Teachers will find that Ss will not learn less, but more. Don't force the use of the calculator; there's nothing worse than teaching Ss how to use a piece of equipment rather than teaching mathematics through the use of that equipment. Teachers must be sold on the idea.

- Plan a reasonable inservice program for teachers.
- Survey carefully available models; don't get caught in fake economy. Buy good equipment like Texas Instrument, Bomar, or Hewlett Packard.
- Design evaluation of effects of calculators.

- Buy a supply of them for the school; use them in the classroom at all levels. They are an excellent tool for teaching meaningful mathematics.

- It is still absolutely essential that the child be able to perform the 4 basic operations by hand. The calculator should not be used in a manner that replaces these basic skills. We would hope it could be used to increase interest in problem solving.

- Introduce them in the upper elementary grades.
- Use them to teach the capabilities of calculators and then branch to computer literacy.
- They should not replace instruction on computation (meaningful) but should be as a supplement.
- Make them generally available when they are introduced.

- We should not be charmed by the advent of calculators. Plan activities which require appropriate use of the calculator and encourage its use when appropriate. We need a careful study of curriculum implications of the calculator and especially of the cheap, programmable calculator of the near future.

- Consideration should be given to acquiring some hand-held calculators that would be made available for students' use at various grade levels.

- Strongly encourage their use especially as an aid to problem solving, i.e. removal of mundane aspects of the generation of precise numerical solutions.

- Do not consider the use/non-use issue as black and white.
- Seriously consider using them in a well-planned program where the calculator is seen as an aid in solving certain kinds of problems and in a way that helps persons look at the structure underlying the mathematical operations and their associated algorithms.
- Beware of the "get-rich-quick" publishing houses who are marketing poorly conceived and hastily written printed materials to be used in a calculator-based program of instruction.

- Think of a hand-held calculator like a typewriter; it is likely to increase the use of arithmetic (just as the typewriter has increased letter writing).
- To ignore the existence of hand-held calculators or to ban their use is counterproductive and counter to reality--hand-held calculators are here to stay.
- Expect curricular consequences; some ideas will become more important, some less.
- Hand-held calculators have limitations in availability, precision, and utility. Do not ignore fundamental skills.

- Try to have available at least 30-35 (full class size) of calculators and instruct in basic operations.
- Encourage student projects in the use of hand calculators.

- I would recommend that a long hard look at what we are trying to do in elementary arithmetic must be taken. We must revise our present expectations of behavior on the part of children in certain grade levels. More emphasis on approximation, estimation, common sense and problem solving must be placed in the classroom activities. Let the calculators do the "busy work" and children start "thinking" about mathematics.

- Include their use in math, science, social studies and other curricular areas.

- I would suggest that they find meaningful ways to use them at all grade levels and accept the fact that they are here to stay.

- Prepare kids to use them (emphasize decimals and estimation).
- Allow use when computation is not the principal goal.
- Encourage children to learn to check work.

- Study their benefits.
- Use them if you find benefit.

- Study the literature thoroughly before using the calculator in class.
- Determine clear objectives for using the calculator.
- Attend workshops and inservice sessions on the use of the calculator.
- Consult NCTM reports and research.
- Request university consultant for demonstrations, ideas, information, etc.

- Use them for calculations after students have understood and mastered the basic operations.

- Allow children to use them to check their calculations.
- Calculators motivate children.
- Neat problems can be illustrated with the calculator, especially in the secondary math and science curriculum.
- Use them with discretion remembering that understanding math is more important than being able to compute.

- I would strongly encourage their use. Many students are turned away from mathematics because of problems in computation. This is true for different ability levels. I believe that the skills in computation will come through use and success combined with interval reinforcement. Skills and concepts should both improve.

- Take advantage of them. Find ways to use them to your advantage but still stress skill at computation, number facility, etc.

- Use them but only after considerable thought is given as to how, when and for what purpose.
- Approach their use on an experimental basis.
- If you don't have a faculty member or supervisor who can guide their proper use, get some consultant help.

- Show kids how to use them, then put them aside for the most part.
- Now and then give a test deliberately using very large numbers and let students solve problems with their calculators. This does provide teachers with an opportunity to encourage children to learn when to +, −, x, ÷ .

- My answer would be dependent entirely on the interest and competence of one or more of the faculty. I think we are not now at a point when one should be directing schools to get involved. I think some individuals and some schools will get started and their experience will be useful to us all.

- Use them freely; provide them for students who don't have them.
- Their use should be discouraged (banned actually) only during lessons on development of computational skill.

- Make available (by school system) in classroom (to be used when appropriate).

- I would endorse its use provided it is used in conjunction with a program which emphasizes why computational procedures work as they do. There is no good purpose served if a student can determine a result but not know when to use a particular process.

- I would recommend to replace the log, multiplication and division algorithms with the calculators and using the extra time to teach algebra.

- Tell them to introduce the hand-held calculator slowly for checking purposes, checking either pencil or paper work or checking estimations prior to computations.
- I would encourage teachers to seriously consider the hand calculator as a standard piece of equipment in the classroom.
- The teachers should be on the alert for new applications of mathematics suitable for hand calculators at all grade levels.
- More time can now be spent in teaching what operations to perform in a problem rather than being preoccupied with the calculation itself (which in many cases is trivial).

- Don't overly promote or deny; this is something (a tool) in the real world.
- Let's learn to use it correctly but let's not assume it's the answer.

- Start teaching estimation, quantitative thinking, as an objective. Devise drill where the task is to show how to estimate the product 3.97 x .26. All we do now is make a pass at this and then scold the child when he makes dumb errors--didn't use his "common sense." But it isn't common. It should be taught. The only place where quantitative thinking is taught as an objective (to be learned) is in slide rule work.
Samples:

Given	Student Shows
.35 x 1700	1/3 x 1800 = 600
73% x 9	3/4 x 8 = 6
.89 x 17.6	1 x 17.6 = 17.6
.43 x .17	1/2 x .18 = .09

- Use them selectively, not to replace learning basic facts and standard algorithms, but to open up the set of reasonable problems and projects kids can pursue.

- Children should be exposed to calculators and learn to operate them. However, some practice in hand computation is necessary both for better understanding and so one is not totally dependent on having a calculator. Calculators are just another tool and do not appear to affect learning significantly.

- Examine curriculum to find new opportunities and situations which are not now available for study in K-12.
- Use the calculator in a positive way as a tool to extend mathematical learning by making traditional work easier. You can solve many more of the traditional word problems using the hand-held calculator.
- Focus can be on process because product (computing) is assured.

- Use them, but stress knowledge of basic facts.
- Stress problem solving skills.
- Stress algorithmic process.

- Use them when appropriate . . . better yet, use them!

- First, find out what they can and cannot do so that you are knowledgeable about them. Experiment with their use in selected classes. Develop a positive school-wide policy governing their use in all classrooms; then encourage their use within these guidelines.

- For elementary and slower secondary students use as motivational and feedback (after applying personal computation). For better secondary students, train in usage, then apply when needed (set up problem and perform computations with calculator). In any case, they should be provided for all students or used by none.

- Provide one for each teacher if they want one from grades 1 or 2 to 12.
- Encourage teachers to try out ideas. If they want more machines, try to provide a few that teachers can share.
- Let's get a resource of ideas before we make any decision.

- They are becoming an increasingly evident facet of our culture. It is folly to ignore them. It is easy to develop some facility with them, and we should orient our students in their use, pointing out their strengths and weaknesses, and giving some problems (with very large numbers) that are suitable for solution on a hand-held calculator.

- Have goals firmly in mind and proceed thereto with common sense.
- Stress estimating skills.

- If they are available, use them.
- If used, then there must be a far greater emphasis on (1) conceptualizing processes and (2) approximation and estimation.
- Use in early grades must be marked with care in a way similar to using any other labor saving device.

- Use them only when they facilitate learning in its most important aspects.
- Make sure students understand each computational process and when it is used.

- Purchase experimental sets and have your best staff experiment with them for a year. As a result, issue guidelines for calculator use.
- Consider implications of allowing students to bring calculators from home for use in math classes.
- Consider implications for curriculum in math. Calculators are a fact of life. Let's not fight them!

- Inform all students of the capabilities and limitations of various calculators.
- Encourage students to obtain calculators commensurate with their needs and finances; do not rush into a heavy investment in rapidly outmoded calculators.
- On all tests (etc.) be sure the students have equal access to calculators. (An emphasis on setting up problems for calculator use is often desirable.)

- Encourage teachers to set up real-life type problem solving situations which require application of the calculator to solve the problem. In other words the problems no longer need to have integer solutions.
- Also visit business places and apply the calculator to everyday situations.

- I would suggest students in the advanced high school science and mathematics courses be encouraged to use calculators in much the same way they did and to some extent still use the slide rule.
- In elementary grades and general math classes, I would propose they be used somewhat the way desk calculators have been used in some instances in the past.

- Buy calculators appropriate to the level of the students and with desired properties (floating decimal point, algebraic logic, no gross errors, etc.)
- Conduct careful inservice programs.
- Prepare guides (or acquire them) for where and how to use them.

- I would not give advice on this topic.

- Study how they can be used to help teach the mathematical concepts and skills we want students to learn.

- Use them!! Properly!! with planning and purposes carefully considered. This requires thoughtfully developed problem situations (e.g. find π, $\sqrt{2}$, $\lim \frac{2n + 3}{n}$, problems where areas change as perimeters are fixed, but dimensions change, etc.) Have enough machines (but students can work in pairs or teams, etc.).

- Ease time doing simple calculations so that more material or richer material can be developed.
- Provide reinforcement of basic facts and algorithms.

- Replacement of algorithmic skill understanding and practice.

- Their use should be justifiable from a learning or educational viewpoint. We do need to research their use in terms of specific outcomes.

- For elementary teachers: Consider limited use until a defensible program is developed.
- For secondary teachers: Participants in conferences, local planning groups and action research, to learn about or contribute to calculator involved curriculum development.

- Encourage their use generally, but emphasize their limitations.
- Be specific in many instructional settings (e.g. use only to verify if you calculated correctly).
- Use to provide immediate confirmation during practice.
- Emphasize estimation skills.

- Allow their use because of reasons given previously (less time would be spent on paper and pencil drill; greater speed and accuracy in computation should result; more time and a means would be available to develop greater understanding of concepts).
- Where students cannot afford their own, provide them as the school would provide other teaching and learning tools.

- Use freely after about grade six for all sorts of checking and computation.
- Also useful for exploring new algorithms.

- Use them in the classroom; however, some skill should be developed in mental computation.
- Use calculator to check computation.

6. *What modifications do you believe should be made in the K-12 curriculum if/when hand-held calculators are made readily available to students at all times? Star the three you believe to be most significant.*

* More attention to estimating.
* More attention to multiplying and dividing by powers of ten.
* More attention should be given to programming problems (translating from familiar to math language).
- More attention must be given to decimals at an earlier point in the elementary school curriculum.
- More realistic problems could be given to students.

- No "modifications" in curriculum as such, but rather the incorporation of the calculator into traditional programs as a means of developing basic concepts, checking answers, estimation, discovery and laboratory approaches, etc. Possible changes: decreased emphasis on fractions, logarithms, use of trigonometry tables and other tables (square root) as well.

* Include problems whose solutions were prohibitive.
* Encourage development of algorithms.
* Increase the number of exercises of a "numerical nature."

* More emphasis on solving real problems.
* Facilitate teaching of computer programming (which is really a subtopic of above).
* Eliminate teaching the slide rule.

- Really haven't thought too much about this; however, I am sure there might need to be "practice exercises" just as there are for "slide rule courses"--or as there were.
- One may wish to decrease slightly practices on calculations "by hand."

- Activities designed specifically for hand calculators must be developed by creative curriculum developers.

* More emphasis on problem solving.
* More emphasis on creative mathematical activity.
* More emphasis on mathematical processes vs. emphasis on products (answers).
- More emphasis on decimal equivalents of fractions.

- Give more emphasis to computation with decimals and less to mixed numerals--mainly because of metrication, not electronic calculators.
- Restrict computation with fractions to "easy" fractions. (Structure needed for pre-algebra considerations.)
- Incorporate more "real" world problems with "real" numbers in the problem solving strand.

* Estimation and approximation needed to mentally check the hand-held calculator's answer.
* De-emphasize fractions, emphasize decimals.
- Examples and exercises need not be devised to work out nicely. Students can make up problems.
- Order of operations needs study.
- Get out of the "back to basics" movement and teach structure with hand-held calculators for computation.

* Kinds of numbers in word problems, especially for general math.
* Do not ignore fractions even with SI and calculators.
* Emphasize estimation.

- A clearer distinction needs to be made throughout the curriculum as to when we are teaching for process as opposed to when we are teaching for product. In the latter case, the use of a calculator will save time which may induce curriculum modifications. In the former, the use of a calculator would not be central.

* There should be an increased emphasis upon problem solving, using the hand calculator as a possible tool to eliminate long, complex calculations.
* There will have to be more emphasis upon decimal form of fractions.
- "Built-in" work on how to use the calculator appears to be a "given."

- At this time I can't see any changes. Recall "teaching machines." The machines were to revolutionize the schools. Students would do most of their work at home. What happened to those machines? Millions of dollars went into experimentation. The Wall Street Journal at one time devoted the front page to them. Where are they? Modifications later--maybe. Maybe, if calculations are speeded up, we may be able to add a few more topics. On the other hand, maybe we can then be sure that all of the single basic competencies are learned.

- Less time devoted to "drill and practice" and more time to problem solving and applications.

* Teach some mathematical functions earlier such as SIN, < N, $\sqrt{\ }$.
* Develop good resources of the applications (problems) for which calculators can be used.
* Carefully consider use of calculators in all classes, especially mathematics, science and social science.
- Revise drill procedures in elementary school.
- Consider teaching "flow charting."

- Teach decimals and estimation; encourage checks; teach to use.

* More problem solving.
- More investigation into number theory.
- More investigation into what might happen if some of the data were changed (mini-mathematical modeling).

* Less time on computation, more time on concepts.
- New material in curriculum, e.g. geometry in elementary grades and possibly probability in elementary grades; these will be possible as a result of the time saved by utilizing the calculator (for example why teach the square root algorithm?).
- After my recent visit to the University of Georgia, I am convinced that transformation geometry will be the next major push in elementary school curriculum.

* As an introduction to computer literacy.
* To teach problem solving.
* General utility in every day life.
- Flow chart logic.

* We can have less concern for skill in standard algorithms.
* Mental arithmetic will take on new importance.
* We must continue and increase our concern for meaning and understanding of process and relationship.
- More realistic problems with large masses of data can be handled.
- We may need to de-emphasize skill development in fraction arithmetic.

* Increased attention to estimation skills.
* More attention to problem solving, less on problems involving messy computation.
- Provides an alternative to developing basic skills.
- Additional attention devoted to decimals.
- Relationships between fractions and decimals.

* Re-orientation of emphasis from computation to higher level objectives.
* Development of curricular materials designed to higher level objectives.
- Encourage estimative procedures to a greater extent.
- Encourage iterative procedures to a greater extent.

* More emphasis on decimal fractions than on common fractions.
* Problem solving, particularly multi-step problems, will probably receive more attention.
* Consideration should be given to using the calculator as a drill and practice device, particularly as a validating technique.
- De-emphasize the long division algorithm.
- Use metric measurements more predominantly.

- Decrease drill and practice in computations.

- More problems with real data.
- More emphasis on problem-solving strategies and algorithms.
- Less drill on certain skills, e.g. long division.
- Earlier introduction of decimals, perhaps scientific notation also.
- Earlier introduction of negative numbers--nothing like "3-5 can't be done."
- Successive approximation, sequences, and limits are easier to do.

- In courses such as probability and statistics, more inductive experiments (to precede deductive arguments).
- More emphasis upon approximations in measurement and study of error.
- More emphasis upon computational algorithms suitable for calculators.

- I believe there are many researchable questions regarding their use in classrooms which should be investigated.
- We must be willing to forego some of our popular assumptions about what and when children of certain ages should "know" about mathematics, i.e. do all entering 4th graders need to have committed to memory all the basic multiplication facts or will they learn them in time through use.

* Can now put more time on the concept of the 4 fundamental operations and less on the algorithms.
- Can make math fit the real world better.

- Less time on practice with the algorithms in paper and pencil practice form.
- More time to develop the explanations as to why and under what circumstances algorithms work.
- More time for applications with a broadened range.

* Greater emphasis should be placed on problem solving.
* Structure should continue to be emphasized.
* Textbook exercises should be revised to include proper use of calculator.
- More enrichment material, employing the use of the calculator, should be devised.

* Shortening amount of instruction with long division, division of fractions, operations with decimals and percents.
* Increase the use of calculators to solve practical problems, i.e. have more practical problems.
* Focus more on concepts, less on computational skills.

- Nothing startling--maybe add units of study inwhich the calculator can be used. I still think computational skills should be stressed.

* More emphasis on rational number concepts and less drill on computation with fractions, particularly fractions related to English system measures.
* More emphasis on estimation and boundary problems.
* Earlier work with sequences, series and limits.
- More emphasis on the decimal system.
- Support metrication.

- At this point I don't foresee a fundamental change in the course of study; that is, the same topics are apt to be taught in the same sequence. But there are apt to be fundamental changes in the approach to topics, the number and kind of functions that can be considered, and an increase in types and kinds of problems to be solved.

- More emphasis might well be spent on teaching children the meaning of the operations, i.e. knowing what operations to perform.
- Estimation could be emphasized more heavily.
- More problems from science dealing with conversion from one unit to another might be employed.
- More computation involving approximate measurement could be performed.

- Heavier emphasis on estimation.
- Change in the nature of problem solving at almost all levels.
- Again, we search for more effective ways to improve understanding, this time through the use of calculators.

- Increased use of learning center or laboratory in math instruction.
- Teach math behind the calculator.
- May be able to add more math content if calculators speed up the process of learning.

* Build in lab activities where computation is involved but the emphasis is on learning other math concepts.
* Use calculators with primary students only briefly for motivational purposes.
* Develop standardized tests which allow for use of calculators.
- Build in more work with no patterns, particularly with greater numbers.

* More emphasis will be placed on analyzing word problems and the sequence of the operations involved in solving a problem.
* Because of the above, there will be a greater need of reading proficiency among the students than they now possess.
* Routine page after page of dull computational problems will be minimized.

- More discovery-laboratory techniques in concept development, especially in higher grades.

* Less emphasis on computational drill.
* More emphasis on understanding and knowing when to compute.
- Integration of mathematics teaching/learning with other subject areas.

- An earlier focus on decimals.
- Problem solving situations can use "real world" data.
- Guess and test strategies for solving word problems will be easier to implement.
- There will be less focus on common fraction computation.
- Drill in computation will be easier to motivate by using the hand-held calculator in conjunction with pencil and paper computation.
- Laboratory situations which involve messy data can be developed using the hand-held calculator to do "on line" calculations.

- Much more emphasis on algorithms, leading to students developing their own algorithms for the calculator.
- Earlier development of decimal fractions.
- New emphasis on (verbal) problem solving.

- Reduce greatly emphasis on computation by logarithms.
- Reduce tabular and interpolation work with trigonometry functions.
- Increase emphasis on estimation of answers (before calculation).
- I'm not very responsive to this question because I don't believe that massive changes would or should be made. I feel that the basic computation algorithms should still be taught and calculators used after that point for motivation, simplification, and applications as appropriate.

* More attention to decimals early.
* Increase problem solving attention.
* Make decisions about "need to know" levels of computation success.
- Less attention to common fractions.
- Increase attention to estimation.
- Change focus on numeration with emphasis on longer numbers.

* We should encourage students to develop their own algorithms for shortening the computational process on their particular machine.
* We should teach a variety of algorithms for calculators (with vs. without memory, functions, etc.).
* We must teach estimating so they can detect large errors made.
- We should teach students how to use them.

- Stress on application of computation to interesting problems.
- Increased emphasis on statistical inference and on combinatorial problems.

- Greater emphasis on conceptualizing.
- Greater emphasis on estimation/approximation.
- Rewriting exercises to make them more realistic.
- Complete reverse in ratio of "word problems" to standard drill exercises.
- There would be greater emphasis on decimal fractions and less on common fractions (although not completely eradicating common fractions as many would believe).

- Less time will be spent on 3d x 3d multiplication and long division.
- Decimals will be introduced earlier in the elementary sequence. Perhaps introduction of metric will force this change also.
- Arithmetic computation, as a factor in accuracy, will nearly totally disappear from senior high math courses.
- Senior high math teachers will be able to assign much more significant problems, when all students have calculators available to them.
- No one has as yet determined the specific impact on the curriculum--it will be significant!

- Calculators should support rather than replace hand calculation.
- Estimation should be given greater emphasis.
- Precision and accuracy should be stressed.
- Irrespective of above: Application should be given greater emphasis and a wide variety of application should be considered.

* More problem solving situations.
* More applications of math now possible with earlier introduction of decimals--coordinates with philosophy of metric system.
* More emphasis on place value to understand numbers.
- Algorithm understanding reached before calculator application.
- Real life solutions instead of more integers in problems.
- Coordinate decimal system with measurement with money in the curriculum.

- The only modification I would make is to include more "real" applications, i.e. having calculators available allows us to deal with problems which do not have small integers, neat solutions, etc. Two minor changes might be: (1) a selection on round-off error and (2) more emphasis on statistics.

- Why state "at all times"? Calculators should be used some of the time, especially when the instructional goal is problem solving (applying principles) and not first of all drill over computation. So used, calculators wouldn't change much of anything except to allow students to do more work more efficiently.

- I don't feel that they will.

- Less fraction work with complicated denominators.
- More problem solving experiences.
- Less use of tedious algorithms after they are mastered.

- Change the place or $\sqrt{\ }$ in the curriculum.
- Eliminate use of answer keys to check papers.
- De-emphasize computation and increase emphasis on concepts.

* More work with decimals, less with fractions.
* More math-science coordination and integration.
* Greater emphasis on problem solving.
- Spend less time teaching long division computation in grades 4-6.
- More exercises involving patterns (of numbers).

- In the elementary and middle schools: (1) instruction in their use, (2) increased emphasis on problem solving (application situations), and (3) increased emphasis on estimation skills.

* Earlier work with decimals.
* Greater emphasis placed on concepts.
* Additional time available for applications.
- Less time and attention given to drill.
- Aid in metric computations.

- They might decrease the amount of time needed to teach computational skills in the elementary school. I don't mean to imply children would need less skill in computation--the hand-held computer might help them gain that skill sooner. I see no other major modifications that would need to be made.

- Changes will have to evolve slowly.
- There will be an emphasis on decimal vs. common fractions.
- There will be an emphasis in problem solving.
- Concepts will be met earlier and unexpectedly by students (e.g. negative numbers).
- Realistic numbers in real life problems will become more common.
- Students will be encouraged to check work, explore new concepts, examine patterns more often.

- Earlier introduction to decimals.
- More applications of math in elementary schools.
- Large numbers introduced earlier.
- Trigonometry manipulations will be easier and introduced earlier.
- Logarithms will be easier and earlier, used more as a tool than as unit of content.

- This would depend on research findings.

- I haven't given this enough thought. I don't believe "major" modifications of the curriculum are needed but effectiveness of time allotments may change and more applications may be usable.

- They are so dependent on the teacher and the situation it is difficult to generalize.

* Less stress on speed skills and drill.
* Great increase on estimating skills.
* More stress on processing problems, problem-solving.
- More stress on understanding of how numeration/calculation works.

- I haven't really thought this through.

- Include problems of greater reality that call for chains of computations.

7. *Assume that a school system (K-12) has moved to widespread use of hand-held calculators throughout the curriculum and testing program. React to each of the following in such a framework:*

True	*?*	*False*		
19	3	56	a.	The mathematics curriculum would need to be modified extensively.
2	1	75	b.	The mathematics learned in elementary schools would be significantly less.
2	5	71	c.	Students would no longer remember the basic facts.
18	8	52	d.	Parents would strongly oppose such widespread use of hand-held calculators.
15	6	57	e.	Elementary teachers would not use hand-held calculators during mathematics lessons.
0	1	77	f.	Students would no longer have an appreciation for concepts such as negative integers, fractions, square roots, etc.
5	5	68	g.	Fractions will no longer be taught in the elementary school.
53	8	17	h.	Students would improve in the ability to estimate quantities.
53	12	13	i.	Students would improve in the ability to solve problems.
0	3	75	j.	Students would lose interest in mathematics.
3	8	67	k.	Students would become lazier than they are.
4	4	70	l.	Students would lose confidence in their ability to handle numbers.
43	8	27	m.	Students would have a greater understanding of concepts such as square root, negative integers, fractions, etc.
48	10	20	n.	Students would gain understanding of the decimal system.
54	14	10	o.	Students would gain interest in arithmetic.

Appendix B:

5. Summarized Responses from Textbook Publishers

The questionnaire (pages B-75 to B-76) was sent to 26 publishers of elementary and secondary school mathematics textbooks; responses were received from 13.

Survey on Calculators Sample: Textbook publishers

1. What do you anticipate to be the extent (in percent of schools) of use of hand-held calculators in schools --

   a. this year? ____________

   b. in one year? ____________

   c. in 2 years? ____________

   d. in 5 years? ____________

   e. in 10 years? ____________

2. Do you plan to develop materials for use with hand-held calculators?

   ___ Yes

   ___ No

3. If yes, how soon will they be available? ____________

4. If yes, what level? (check any which apply)

   ___ primary (K-3)

   ___ intermediate (4-6)

   ___ junior high (7-9)

   ___ senior high (10-12)

5. If yes, what type? (check any which apply)

   ___ included in textbook

     ___ integrated

     ___ alternative

   ___ supplementary

     ___ workbook

     ___ other -- please specify: ____________

6. If yes, what functions/features of hand-held calculators will be assumed?

   a. at primary level ____________

   b. at intermediate level ____________

   c. at junior high level ____________

   d. at senior high level ____________

7. If you are not planning to develop materials, why not?

____

____

____

8. What modifications do you believe should be made in the K-12 curriculum if/when hand-held calculators are made readily available to students at all times? Star the three you believe to be the most significant.

____

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9. Assume that a school system (K-12) has moved to widespread use of hand-held calculators throughout the curriculum and testing program. React to each of the following in such a framework:

True	False	
___	___	a. The mathematics curriculum would need to be modified extensively.
___	___	b. The mathematics learned in elementary school would be significantly less.
___	___	c. Students would no longer remember the basic facts.
___	___	d. Parents would strongly oppose such widespread use of hand-held calculators.
___	___	e. Elementary teachers would not use hand-held calculators during mathematics lessons.
___	___	f. Students would no longer have an appreciation for concepts such as negative integers, fractions, square roots, etc.
___	___	g. Fractions would no longer be taught in the elementary school.
___	___	h. Students would improve in the ability to estimate quantities.
___	___	i. Students would improve in the ability to solve problems.
___	___	j. Students would lose interest in mathematics.
___	___	k. Students would lose confidence in their ability to handle numbers.
___	___	l. Students would gain understanding of the decimal system.

1. *What do you anticipate to be the extent (in percent of schools) of use of hand-held calculators in schools--*

a.	*this year?*	.01%	50%	1%	5%	5%	50%
b.	*in one year?*	.1%	55%	2%	10%	10%	60%
c.	*in 2 years?*	1%	60%	4%	15%	15%	80%
d.	*in 5 years?*	10%	75%	25%	40%	20%	90%
e.	*in 10 years?*	100%	90+%	80%	80%	70%	100%
a.	*this year?*	1%	1%	10%	5%	5%	1%
b.	*in one year?*	2%	5%	15%	10%	10%	2%
c.	*in 2 years?*	10%	10%	20%	20%	20%	5%
d.	*in 5 years?*	20%	40%	40%	30%	40%	40%
e.	*in 10 years?*	40%	80%	50%	40%	80%	60%

2. *Do you plan to develop materials for use with hand-held calculators?*

Yes: 11 *No:* 2

3. *If yes, how soon will they be available?*

1976; April 1976; Fall 1976; 1978; 1978-1979; 1980-1981; 1985; 1 or 2 years; not definite.

4. *If yes, what level?*

primary (K-3):	3
intermediate (4-6):	9
junior high (7-9):	10
senior high (10-12):	5

5. *If yes, what type?*

included in textbook:	8
integrated:	4
alternative:	5
supplementary:	8
workbook:	7
module mini-course:	1
kit:	1

6. *If yes, what functions/features of hand-held calculators will be assumed?*

a. at primary level
- algebraic
- not decided at this point in time
- checking results
- simple, durable
- +, -, x, ÷
- addition and subtraction
- not certain at this time

b. *at intermediate level*
   - +, -, x, ÷
   - algebraic system, chain and mixed calculations, stored constant, floating decimal point, CE (clear error) key
   - algebraic
   - helping to develop algorithms, checking results
   - algebraic logic, no memory, constant
   - +, -, x, ÷, $\sqrt{\ }$
   - addition, subtraction, multiplication and division
   - 4 basic operations plus percent

c. *at junior high level*
   - +, -, x, ÷
   - algebraic system, chain and mixed calculations, stored constant, floating decimal point, CE (clear error) key, square root
   - algebraic with memory
   - being able to use real numbers in problem solving
   - algebraic logic, no memory, constant
   - +, -, x, ÷, $\sqrt[n]{\ }$, N^n, floating decimal
   - addition, subtraction, multiplication, division, square root, exponential (a^x where a is rational number)
   - as aids in performing some forms of mathematical processes
   - 4 basic operations, percent

d. *at senior high level*
   - +, -, x, ÷
   - algebraic system, chain and mixed calculations, stored constant, floating decimal point, CE (clear error), key, trigonometric functions and a limited memory
   - algebraic with memory
   - time saver, more realistic data, good for approximating roots
   - +, -, x, ÷, $\sqrt[n]{\ }$, N^n
   - +, 1, x, ÷, trigonometry, ℓ^x, π, and one or more memory banks
   - as aids in performing some forms of math processes
   - 4 basic operations, memory, scientific notation, x^y, trigonometric functions

7. *If you are not planning to develop materials, why not?*

Only two respondees were not planning to develop materials. They responded:
- Do not believe they will be used to any extent.
- Not into math right now.

8. *What modifications do you believe should be made in the K-12 curriculum if/when hand-held calculators are made readily available to students at all times? Star the three you believe to be the most significant.*

* Earlier introduction to place value.
* Earlier introduction to decimal fractions.
* Greater emphasis on decimal fractions.
- Greater emphasis on estimation.
- Greater emphasis on problem solving.

* More emphasis on estimating.
* More emphasis on problem solving.
* Realistic problems--not contrived for neat calculations.
- More emphasis on quantitative thinking and analysis, and place value.
- Earlier work with large numbers, decimals, irrational approximations, integers.
- Greater use of mathematical properties.

- Development sequences will change to reflect the capacity to look quickly at patterns.
- Increased emphasis on decimal numeration.
- Increased use of guess and test.

- None--instruction in math classes.

* Emphasis will be on the structure of algorithms rather than on the execution.
* Applied problems can be more realistic.
* Flow charts will become important in order to organize information before using the calculator.

- You would only need to add a special topic within existing structure of math and business classes.
- Could be handled by teachers or by a special supplemental booklet with directions and practice examples.

* Emphasize problem-solving skills.
* De-emphasize routine calculations (paper-and-pencil).
* Emphasize rounding, estimation, checking for reasonableness.
- Introduce more realistic problems involving messy data.
- Minimal use of calculators at primary level.
- Increase work with number theory, exploring relations between numbers, etc.

- Decreased emphasis on computational skills.
- Increased emphasis on problem solving strategies.
- Increased emphasis on estimating and error checking.
- Increased emphasis on manipulating numbers.

- Simply as enrichment concerning existing processes.

* Students will deal with applications (some practical).
* Students will deal with career mathematics.
* Students will be free of computational drudgery and will know when to compute rather than just how to compute.
- Students will deal with dilemma (choice) situations.

* K-6 Less on particular algorithms to be memorized and more on methods.
* 9-12 Little (if any) hand computation.
* K-12 More story problems.

- Few modifications.
- More emphasis on problem solving.

- Special exercise booklets.
- More emphasis on problem solving.

9. *Assume that a school system (K-12) has moved to widespread use of hand-held calculators throughout the curriculum and testing program. React to each of the following in such a framework:*

True	*?*	*False*		
2	0	11	a.	The mathematics curriculum would need to be modified extensively.
1	0	12	b.	The mathematics learned in elementary school would be significantly less.
2	0	11	c.	Students would no longer remember the basic facts.
5	1	7	d.	Parents would strongly oppose such widespread use of hand-held calculators.
2	0	10	e.	Elementary teachers would not use hand-held calculators during mathematics lessons.
0	1	12	f.	Students would no longer have an appreciation for concepts such as negative integers, fractions, square roots, etc.
1	1	11	g.	Fractions would no longer be taught in the elementary school.
9	1	3	h.	Students would improve in the ability to estimate quantities.
9	1	3	i.	Students would improve in the ability to solve problems.
0	0	13	j.	Students would lose interest in mathematics.
1	0	12	k.	Students would lose confidence in their ability to handle numbers.
9	0	4	l.	Students would gain understanding of the decimal system.

APPENDIX C

TEACHING MATHEMATICS WITH THE HAND-HELD CALCULATOR

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You walk into the grocery store and see people using the hand-held calculator to make decisions about what items are the best buys. You ask for an estimate for new carpeting for your home and the estimator arrives with a calculator in hand. You visit your lawyer's office to get advice on your income taxes and there is a hand-held calculator. More and more people are using numbers to make decisions because the hand-held calculator is available.

Estimates for the number of calculators sold in the last three years range from 20 million to 40 million. Mathematics teachers all over the country are beginning to ask how they can use the calculator to teach mathematics better. Students are beginning to bring their calculators to class and are asking if they can use the calculator to do their homework and tests. It is time that mathematics educators recognize that the calculator is an inevitable part of our daily lives.

Now is the time to consider the effect of the hand-held calculator on the curriculum. Think of what would happen in a fifth-grade class if every student had a calculator when the class is studying multiplication. A lesson that normally takes a full class period would be reduced to a ten-minute experience.

To avoid the "future shock" of the calculator it is time that every teacher begin to prepare for the future. Imagine the effect on learning of mathematics if teachers are unprepared for the flood of materials that will soon be forthcoming to fill the vacuum being generated by the hand-held calculator. All of a sudden the teacher must make judgments in terms of what materials are appropriate and that judgment must be made without any experience using the calculator. Preparation of a teacher can begin with one machine. One machine will help the teacher build experience upon which future judgment can be based.

Recent years have provided similar situations. The mistakes made in these situations should give educators some guidance in preventing the future shock generated by the hand-held calculators. The growth and demise of the language lab might be a good example to consider. When language labs were first available, they were put into schools across the country. They were to be a solution to the problem of teaching language. Where are these laboratories today? In many cases, they are gathering dust and taking up needed storage space. Why did they fail to produce the anticipated results? It seems apparent that they failed because of the lack of software support. There are people today who think that the calculator alone will provide the needed answers for the problems in mathematics education. But again we shall find that unless imaginative software is carefully developed, five years from now the calculators will be gathering dust as well. It may be that the calculator's main advantage is that they take less storage space.

There is one very important difference, however; everyone will have a calculator. The point we should learn from the recent history of the language lab is that a fine technological tool without adequate support in the form of software is doomed.

Probably the best example is the dilemma evidenced by what has happened with the "Modern Mathematics" movement. A tremendous effort was mounted to affect the mathematics curriculum. Many good materials were developed, but these materials were buried in an avalanche of poorly conceived materials. They were modern in appearance, but were often traditional in basic concept. This mixture didn't come up to expectation.

All of these movements have lessons to offer as we look at the beginnings of a new movement that is being generated by the invention of the hand-held calculator. We can expect to make many of the same mistakes, but this time there is a big difference. The calculator will be a part of the lives of every student regardless of what we do about the curriculum. For this reason, the calculator provides another opportunity to generate a mathematics curriculum that will meet the needs of the future adults.

Let us look at some of the general changes we can anticipate as a result of the calculator.

In the beginning we will see two basic approaches. There will be some who want to put units of study in the curriculum at various levels to teach the use of the calculator. Actually, there is very little need to teach the use of the calculator. If you give calculators to ten-year-olds, they soon find out how to add, subtract, multiply, and divide. A little experimentation by the student is all that is all that is necessary. The student may soon be convinced that all pencil-and-paper activity with mathematics is unnecessary. Although there are more complex uses for the calculator, these uses make little sense unless they are supported with conceptual know-how.

The second approach will be more common. Teachers will use the calculator as they would other teaching aids that support the objectives of the curriculum. You can better understand the relation between common fractions and decimals when you have the calculator to generate the examples. You can better teach estimation when you have the calculator to verify the estimate.

Out of this second thrust in using the calculator, you can expect people to become aware that the basic curriculum will be more effective if the materials are designed with the calculator in mind. At this point a danger exists. Decisions will be made in terms of the relative importance of the various topics and the whole curriculum will be revised. Let us hope that by this time we have developed some guidance from research and experimentation to help make wise choices.

THE ROLE OF THE CALCULATOR IN THE PRE-COLLEGE CURRICULUM

Our position on the role of the calculator in the curriculum is a function of the age of the student and the principal objectives of the mathematics content for each level. The following paragraphs summarize our position by level.

1) The Primary Level (K-3) Incidental use is recommended. One or two calculators in the mathematics interest corner would provide an opportunity for both the teacher and the student to explore its use. If dangers exist they are at this level. All students should be guaranteed the opportunity to develop the basic mathematics skills with pencil and paper.

2) The Intermediate Level (4-6) Each elementary school should have a classroom set available for occasional use to support the present objectives of the curriculum. Each teacher should also have one or two to use on an individual basis and to support classroom activities.

3) The Junior High (7-8) A classroom set should be available for each Junior High teacher. The calculator should be used whenever it is the best way to develop the objectives of the curriculum.

4) The Senior High (9-12) Every student should have a calculator available anytime one is needed.

The following pages give examples of the use of the calculator in relation to the principal content objectives for each level.

THE PRIMARY LEVEL

The principal function of the primary years is to build the images and symbols for numeration and the fundamental operations with whole numbers.

Because the fundamentals are built at this level, we feel that the greatest danger in using the hand-held calculator exists here. We would recommend one or more calculators be in the classroom and that their use be incidental to the basic program. Students would use the calculator to verify, extend or explore, but the use would be in an individual or small-group mode such as would occur as a part of a mathematics interest center. Seldom, if ever, would it be a total focus for the class. Such use could be supported by activity cards such as these.

Activity Card Grade 1

Make the calculator count. How far can you count in a minute? Find a way to make your calculator count by 2's, 5's and 10's.

Activity Card Grade 2

Enter the number of tens, push $\boxed{+}$.

32 $\boxed{+}$	125 $\boxed{+}$
45 $\boxed{+}$	241 $\boxed{+}$
96 $\boxed{+}$	643 $\boxed{+}$
43	105
Push $\boxed{+}$	Push $\boxed{+}$
Check 20	Check 10

Activity Card Grade 3

Find two numbers whose sum is 77 and whose difference is 11.

There are many games that would be appropriate for second or third grades. Here is one that is fun.

Activity Card Hit the Target

Push one digit key. Push $\boxed{+}$. Hand the calculator to the second player. Push any digit key. Push $\boxed{+}$.
The first player to display 50 is the winner.

The calculator can be used for drill as you would flash cards. For example, for addition, the student enters $\boxed{3}$ $\boxed{+}$ $\boxed{5}$ gives the answer and then pushes $\boxed{=}$ to verify the answer.

Sometimes the calculator can be used to support or extend a concept. For example, the third-grade class has been studying area. Today the class is organized into teams. Each team is measuring the door, or a window, or the chalkboard to determine the area. Since the final computation to determine area is beyond the students' skills, it is done on the calculator. The concept of area has been built through a variety of experiences, but is culminated and extended by using the calculator.

Perhaps after careful research, it may be possible to use the calculator to help students build the basic images for the operations much as Cuisneaire rods are used today. But using the calculator extensively at this level should be done cautiously. It is very difficult to determine the effect of a substantial curriculum change with small children. It often takes years before the final results are known.

THE INTERMEDIATE LEVEL

The principal objective of the intermediate grades is to develop the manipulation of the whole numbers and the positive rationals. All students should be guaranteed the opportunity to develop "need to know" levels of manipulation with pencil and paper. We should recognize that the existence of the calculator has affected these "need to know" levels. All students should be able to add or subtract any pair of three-digit numbers, to multiply any pair of two-digit numbers and to divide two-digit numbers into any four-digit number. When a student has accomplished this goal with pencil and paper he or she should be able to develop the skill of handling larger numbers using the calculator.

At the intermediate level, a few calculators should be available in all classrooms. They should be used to support the development of the objectives of the program. Because of the calculator, the current objectives should undergo certain modification. To better understand the role of the calculator, we will consider the effects through examples from various strands in the curriculum.

NUMERATION STRAND. Many of the concepts taught in the numeration strand can be supported by calculator activities.

Activity Card

nine thousand thirty-five ⊞
fifteen thousand nine ⊞
thirty-five thousand two hundred ⊞
two hundred thousand ⊞
seventy-two thousand three hundred ⊞

CHECK NUMBER 331544

In this activity experiences in reading large numbers are provided by having one student read the numbers and a second student enters the numbers in the calculator and adds. The input is checked by comparing the total with a check number.

Experiences with large numbers are also readily available using the calculator. For example, after experimentation in counting, the students estimated 24,260 grains of sand in a teaspoon. Using the calculator and their estimate, they determined an estimate for the number of grains of sand in a gallon pail. In a similar experience, they found the number of times their heart beat in a minute and estimated the number of heart beats since they were born. The calculator can contribute to the use and meaning of large numbers.

There are many ways to use the calculator to support the development of numeration skills. The place value concepts are obvious using the machine and there is no danger in using it in this mode.

COMPUTATION STRAND. Once the student has developed the basic levels of computation skill, there are many ways that a calculator can be used to facilitate the understanding of computation algorisms.

For example, the student could be asked to use the calculator to fill in the "holes" in these algorisms:

			23
9◯	891	46	54 ⟌1205
◯2	-2◯	3◯	1◯
125◯	616	2◯0	165
265◯		138◯	16◯
		1610	

Activities such as these can be used to teach the important computational ideas without interfering with pencil-and-paper skills.

The calculator can often be used to effectively develop generalizations. For example, it is easy to develop the zeros generalization when the student uses the calculator to multiply numbers by 10 or by 100. A few examples with the calculator replaces many examples with pencil and paper.

The calculator increases the need for estimation skills. Even though a calculator user quickly learns to visually check each entry, mistakes are made. A good estimator can usually recognize when these mistakes are made. To increase the student's estimation skill, the teacher can give an example orally, like 65 x 42. Each student makes an estimate and one student checks the estimate on the machine.

Many games can be played on the calculator that also increase the estimation skill. For example, try this with a partner.

Hit the Target

Target

490 —— 510

Start Number 15

Operation $\boxed{X}$

First player: Enters 15, pushes $\boxed{X}$
Enters estimate, pushes $\boxed{X}$
Hands the machine to opponent.

Second player: Uses display from opponent.
Enters estimates, pushes X
Returns the machine to first player.

Play continues until someone displays a number between 490 and 510.

As the calculator becomes more common, the need for mental computation increases. The calculator can be used to support mental computation practice in the classroom. The teacher or a student gives a series of examples for mental practice like:

$$(6 + 10 - 3 + 7) \times 4$$

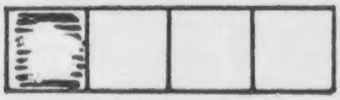
The students compute mentally. One student checks the answer on the calculator.

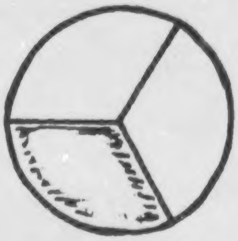
We believe that the proper use of the calculator can increase the student's computation skills and understanding.

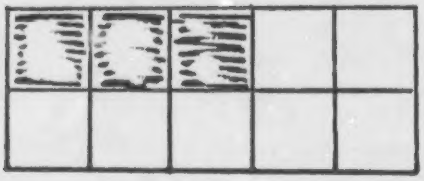
RATIONAL NUMBERS The existence of the calculator coupled with the change to the metric system should have more effect on instruction in use of the rational numbers than on any other part of the curriculum. The minute students pick up a calculator, they are operating with decimal fractions. The only question is how long it will take mathematics educators to adjust the curriculum to respond to this change. Will we continue to spend weeks each year teaching operations with rational numbers in $\frac{a}{b}$ form or will we increase the time with decimal fractions and their operations? Only time will answer this question.

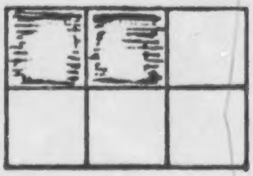
If students can operate with whole numbers using the calculator, they can operate with decimals as well. The problem becomes that of making sense out of their answers. Students should have experiences with the usual images for fractions by responding with decimal fractions.

For example, use your calculator to determine the decimal fraction that is shown:

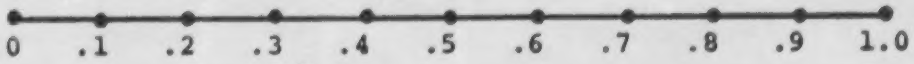








They should have similar experiences with the number line. For example, use your calculator to locate points on the line for these fractions:



☆ for $\frac{1}{3}$ ○ for $\frac{5}{8}$

□ for $\frac{3}{5}$ △ for $\frac{8}{9}$

Much of the emphasis should be on interpreting the decimal answers so they make sense. For example, when sixth graders were answering a problem dealing with the number of buses needed to transport the students of the school on a field trip, their answer was 17.417215 because that was what they got on the calculator. When asked who was going to drive the .417215 bus, they decided that they needed 18 buses. The calculator puts a focus on interpreting the answers.

Of the time today's student spends on fractions, about 80% is spent on fractions in the $\frac{a}{b}$ form and only about 20% on decimal representation. Since the students of the future will use the calculator for most computation and the metric system for measurement, the time allotments should be reversed. The student will always need images for simple fractions, but most computation will be done with decimal fractions.

The calculator's main advantage is in the fact that you can generate answers to many examples rapidly and accurately. For example, machines with the facility to use a constant multiplier can be used to generate a sequence by multiplying by the same decimal.

$$\begin{aligned}
.4 \times 200 &= 80 \\
.4 \times 20 &= 8 \\
.4 \times 2 &= .8 \\
.4 \times .2 &= .08 \\
.4 \times .02 &= .008
\end{aligned}$$

The student's intuition and understanding of the situations when multiplying by decimals can be increased through this kind of calculator activity.

The machines with floating decimal points make it easy to identify the usual difficulties with ragged decimals in addition and subtraction. The student who has 31.7 as an answer when adding 17.2 + 1.45 better understands the difficulty when he or she finds the answer is 18.65 on the calculator.

A difficulty with decimals has always been the placement of the decimal point and the awareness of what the digits mean. The machine can be used to focus on these difficulties. It is important to remember that on the machines, the student has no more difficulty with operations on decimals than on whole numbers. This allows the teacher to put the emphasis where it belongs.

MEASUREMENT The use of the metric system and the calculator opens up a whole new dimension in teaching measurement.

For example, it is advantagous for the student to find the area of at least one triangle by measuring all three heights and all three bases and determining the area in three ways. In the English system and when computing with fractions, this is difficult if not impossible. If you take the measurements in centimeters and compute with the machine, the process is easy and the generalization comes home to the student.

For a second example, consider the ease of finding the volume of any rectangular box if the dimensions are measured in the metric system and the computation is done on the machine. A whole new approach is possible with the metric system and the calculator. The experiences not only can deal with real data, but can be more efficiently handled by combining those two resources.

PROBLEM SOLVING The major reason to teach mathematics is to solve problems. Today most problem-solving activities are further experiences in computation. This is primarily because problem-solving follows a computation unit. The problems are more practice for the computation just taught. Students should study problem-solving as a skill. This study should occur over a substantial time period with a variety of situtations where decisions as to how to solve the problem are necessary.

The calculator eliminates many of the difficulties the students have with the usual problems that are the computational type. For example, with a group of sixth graders, we prepared a set of 40 problems from various textbooks. We gave the set and calculators to 28 students. At the end of a 20-minute period, the students had completed an average of 27 problems and almost all of the answers were correct. If problem-solving skills are related to the number of problems the students can solve, the calculator can be a real help in problem solving.

Most of the problems that are available in text books are not representative of problems in the "real" world. The data are necessarily artificial to limit the computational difficulty in the problems and generally these problems are uninteresting to the students. More interesting problems are now feasible.

> How high would a stack of 20 billion hamburgers sold by the hamburger chain be if they were piled one on top of another? If you can make 600 pounds of hamburger from one cow, how many cows were required to produce the hamburgers?

Mini projects involving experimentation, data collection, and real problem solving can provide worthwhile activity when the calculator is used. One example of a mini project is illustrated.

A teacher brought a bathroom scales, four small wooden blocks, and a jump rope into the classroom. The students watched couriously as the teacher positioned a block under each corner of the bathroom scales and, after having slipped the rope under the scales, proceded to step on the scales and pull up on the ends of the rope. This measured the teacher's pulling strength. Most of the students were anxious to try the experiment themselves. They quickly got into a discussion on how to compute the number of kilograms a person could pull. They agreed if a person weighed 44 kilograms and could pull on the ends of the rope hard enough to make the scales read 79 kilograms, that person would have a pulling strength of 35 kilograms.

After the teacher demonstrated the proper way to pull on the ends of the rope, students performed the experiment and recorded their weight and number of kilograms pulled. After the data were collected on a bulletin board chart, students were interested in determining who were stronger, the boys or girls. With the aid of hand-held calculators, the class computed the average pulling strength of boys and girls. The boys' average was less than the girls', so the boys wanted to try the experiment again.

One of the boys thought that the experiment might not be fair, since bigger students should be able to pull more than smaller students. This possibility was discussed and it was decided that the ratio of student weight to their pulling strength could be used to make comparisons. Again the calculators came in handy. These ratios, expressed as decimals rounded to the nearest hundreth, were recorded on the bulletin board chart. Students then computed the average of the ratios. Again the boys lost.

The boys left that day determined to spend time that evening thinking about how they could use the calculator and the data collected to prove that they were indeed stronger than the girls.

The calculator should not replace the basic skills experiences in the intermediate grades. It is not necessary to have a calculator for every student. A few calculators or even one can add a real dimension to the math activities. At this time, we recommend that the calculator be used to support the development of the objectives of the present curriculum. As people get more experience, we expect modification to take place that will make the calculator even more functional, but those modifications should be made slowly and with care to assure the basic learning for all students.

JUNIOR HIGH LEVEL

The purpose of junior high mathematics has not been agreed upon. Some teachers have felt the purpose is to complete the elementary programs and guarantee that all students can successfully compute. Other teachers have felt its purpose is to prepare students for the study of more advanced mathematics. Neither of these views is entirely satisfactory to junior high students themselves. Junior high students are active, curious learners. They are at the age when they are seeking a purpose and reason for the learning of mathematics. They want an answer to the question, "What can I use this for?"

The hand-held calculator as a standard piece of classroom equipment, has a role to play in giving junior high mathematics a new focus independent of both elementary and secondary school mathematics. Having calculators available in the classroom provides all junior high students a means of performing arithmetic computations. This removes arithmetic proficiency as a prerequisite to application and extnesion of mathematical ideas. Student access to the calculator also means it is possible to provide experiences in which studdnts can perceive mathematics in real and meaningful situations. Therefore, with the calculator, the junior high mathematics program should focus on making mathematics useful for all students. To better understand these effects, we will consider the role of the calculator in regard to various strands of the program.

THE ROLE OF COMPUTATION Perhaps the greatest consequence of the calculator on the junior high mathematics program is that it equalizes students' ability to perform computations. It becomes obvious in the classroom that with a calculator, all junior high students can perform computations with speed and accuracy. The exercises below, for example, can be computed with a calculator in a matter of seconds.

$$\begin{array}{r} 36{,}725 \\ \underline{+\ 4{,}029} \end{array} \qquad \begin{array}{r} 3265 \\ \underline{\times\ 427} \end{array} \qquad 356\overline{)40296} \qquad \begin{array}{r} 370{,}706 \\ \underline{-\ 54{,}894} \end{array}$$

The same exercises utilizing paper-and-pencil methods would take as long as 10 minutes for some students and an eternity for others. With a calculator, exercises of the type shown serve a very limited purpose, other than calculation for calculator's sake.

While the need to perform heavy computation with paper-and-pencil methods diminishes with access to a calculator, the need to know when to add or multiply increases. What now become important are questions such as, how can I use my machine to help me solve this problem and what does my answer mean.

Although the calculator removes the burden of tedious computation, it will not replace the need for quick recall of basic facts, proficiency in simple paper-and-pencil computations, or the ability to make good estimates.

When using a calculator, the ability to make good estimates becomes increasingly important. Mistakes are sometimes made. Wrong numbers can inadvertently be entered into a calculator. Being able to sense the reasonableness of displayed computations becomes desirable. Many text book drill-and-practice exercises can be modified to become interesting estimation activities. Pairs of students, for example, can use a calculator and a set of division exercises to play this mental estimation game.

44184 ÷ 56
57426 ÷ 102
424742 ÷ 53
60507 ÷ 486
99780 ÷ 12
378048 ÷ 88

1. The game starts by selecting one of the division exercises. Each player has 30 seconds to secretly write down what he or she thinks the quotient is.
2. Then use the calculator to see who came closer to the actual answer.

3. The player whose estimate comes closer wins the round.
4. The first player to win 10 rounds is the winner.

The impact of the calculator on junior high mathematics programs is that all students can perform basic computations on an equal footing. The effects of the calculator on content strands in junior high programs are discussed in the sections which follow.

RATIONAL NUMBER STRAND. The calculator and the increasing emphasis on metrication will decimalize the rational numbers and may well spell the end to complicated computations performed with rational numbers of the form $\frac{a}{b}$. Students who have access to a calculator are constantly operating with decimal names for rational numbers. It is common to see students who are playing with their calculator perform operations of the types $-4.1 \times 3.2 = -13.12$ and $-2.1 + 67 = 64.9$. This results in exposure to positive and negative rational numbers prior to formal classroom introduction of the topic.

The decimalization of the rational numbers means less time devoted to fractional numbers and more time to building decimal images, understanding decimal operations, and applying decimals to problem situations. The calculator can be used in a variety of ways to accomplish these objectives. Having students make their calculator count by tenths gives them a feeling for the ordering and relative size of decimals. Sequences of calculator exercises like these, can help students make generalizations on how to perform paper-and-pencil as well as mental operations with decimals.

1250. ÷ 250. = _____
125. ÷ 25. = _____
12.5 ÷ 2.5 = _____
1.25 ÷ .25 = _____
.125 ÷ .025 = _____

Problem-solving settings can be created to build a feeling for positive and negative rational numbers.

> Find a way to use your calculator to show what the temperature will be every hour if the temperature now reads 15°C and starts to drop 4.3° every hour for the next 8 hours.

Applications of decimals to real world problems are often more motivating when a calculator is used. Students can, for example, measure the thickness of all the pages of a book, determine the number of pages measured, and then use a calculator to compute the thickness of a single page. When students use only paper-and-pencil computations, they seldom attempt to verify or extend their answers. When calculators are used, it is common to observe students measuring other books to see if there is a variance in the thickness of pages in different books. Some use their calculators to extend the problem by exploring how many pages would be needed to make a stack of paper as high as they are tall.

By taking advantage of the decimalization of the rational numbers, teachers should have greater opportunities to create problem situations in which students can apply their understandings of the rational numbers. The similarities between computation with the whole numbers and computation with decimals should make the application of rational numbers to problem situations easier for students.

NUMERATION STRAND. Many of the concepts taught in the numeration strand take on new importance when students have access to calculators. A team of students, for example, who wanted to use their calculators to compute the number of kilometers that light travels in a year, had to cope with the limitations of their calculators to display the product of 299,800 (about the number of kilometers light travels per second) times 31,536,000 (number of seconds in a year). These students developed a renewed appreciation of our base ten system when they realized they could do 2998 $\boxed{x}$ 31536 $\boxed{=}$ 94544928 and then annex five zeros to write the distance as 9,454,492,800,000 kilometers.

Exposure to very large and small numbers are frequent experiences for students using calculators. In their desire to handle large and small numbers, they see a need for a working understanding of such ideas as exponents and scientific notation. There are many calculator activities which support this understanding. For example, students visually get a feeling for exponents and exponential growth by entering a number greater than one and then repeatedly pushing the $\boxed{x}$ key. Entering 10 and then repeatedly pushing the $\boxed{\div}$ key displays the decimal names for 10, 1, $\frac{1}{10}$, $\frac{1}{10^2}$, $\frac{1}{10^3}$, $\frac{1}{10^4}$, and so on. Entering 10 and counting the times the $\boxed{x}$ button is pushed gives additional meaning to why $(10^3)(10^4)=10^7$. Finding a way to use the calculator to multiply 367,000 x 37,420,000 prepares students for $367 \times 10^3 \times 3742 \times 10^4$.

By using their calculator, students gain confidence in their ability to handle and interpret large numbers.

A recent publication stating that for every baby born in the United States, our planet will eventually have to provide:

210,000,000	liters of water
79,000	liters of gasoline
22,330	kilograms of meat
220,000	kilograms of steel
1,000	trees

took on a greater impact when students multiplied these figures by the estimated annual increase in the population of the United States, 3,759,500. Using their calculators, students realized this means an eventual claim on our planet for:

789,495,000,000,000	liters of water
297,000,000,000	liters of gasoline
83,949,635,000	kilograms of meat
827,090,000,000	kilograms of steel
3,759,500,000	trees

Analyzing real data with the help of a calculator is an excellent means of helping students acquire a firmer grasp of how mathematics is useful in interpreting events around them.

METRIC GEOMETRY STRAND. The use of the metric system and the calculator should make the study of selected topics in geometry more interesting for students as they apply measurement and computation ideas to finding out about the world they live in. Decimal names for rational numbers and the ability to compute with real world data means students can experimentally develop many geometric concepts and relationships.

For example, students with metric tapes can measure the circumference and diameter of tin cans and then use their measurements to compute an approximation for pi. With the aid of a calculator, they can easily perform this activity for many different size cylinders, making the generalization of how circumference is related to diameter more obvious. Metric tapes, calculators, and an assortment of containers are pieces of equipment for activities to replace flat textbook drawings. Making a game out of first estimating the volume of cardboard boxes and computing the volume is another example. Yet another example is having students investigate how doubling the height or radius affects the volume of a cylinder.

PRE-ALGEBRA STRAND. The calculator should have an effect on how many pre-algebra topics are taught and learned.

Most junior high students are familiar with the "machine" idea of a function as seen in textbook pictures and diagrams. Students who enter a number and then push $\boxed{x}$ 3 $\boxed{=}$ see that given a certain input, the calculator can display only one output, and that the output depends on the choice of the input. Using the calculator code $a\ \boxed{x}\ 3\ \boxed{=}\ b$, students can enter large and small numbers in the calculator and quickly generate a set of (a,b) ordered input-output number pairs.

A variety of calculator codes, both linear and non-linear, are possible for students to investigate.

$$a\ \boxed{+}\ 3\ \boxed{x}\ 7\ \boxed{=}\ b$$
$$a\ \boxed{x}\boxed{+}\ 3\ \boxed{=}\ b$$

The graphing of functions also becomes more exciting with the use of the calculator. Students can put random numbers in the calculator for $a\ \boxed{x}\ 3\ \boxed{=}\ b$ and see if the ordered pairs result in points on the same graph. These calculator graphing experiences lead naturally to a discussion of the domain and range of the function.

Once students have calculator code experiences, they are ready for a more traditional algebraic representation of the functions. With a little experience, students can easily decipher $a\ \boxed{\div}\ 3\ \boxed{=}\ b$ as $\frac{a}{3} = b$ and $a\ \boxed{x}\ 3\ \boxed{+}\ 7\ \boxed{=}\ b$ as $3 \cdot a + 7 = b$.

Traditionally first experiences with equations use the basic +, -, x, and ÷ operations with whole numbers. With the calculator, students who are asked to determine the input number for $a\ \boxed{\div}\ 3\ \boxed{+}\ 5\ \boxed{=}\ .085$ are forced to use the "doing" and "undoing" ideas of solving equations. By having already experienced generating output numbers (the doing process), they are quick to realize that $.085 \boxed{-} 5 \boxed{\times} 3 \boxed{=} a$ (the undoing process) is the calculator code for solving the equation $\frac{a}{3} + 5 = .085$.

Attempting to use a calculator code to solve $3 \cdot a + 7 \cdot a = .45$ can lead students to rewriting the equation as $10 \cdot a = .45$, thus giving a fresh approach to their understanding of the properties of rational numbers.

The use of calculator codes may seem like an unorthodox and even alien approach to pre-algebra topics at the junior high level. But for students who are still in the process of developing their mathematical experiences, the calculator can make finding out about such topics as functions, graphs, and equations an exciting and meaningful experience.

PROBLEM SOLVING. A major purpose of junior high mathematics is to develop a student's problem-solving behaviors.

Typically the student's exposure to problem situations is a textbook word problem in which the major task is to determine the appropriate mathematical operation--should I add, subtract, multiply, or divide with the numbers? Since textbook problems usually appear at the end of a set of computation exercises, they are often more like a computation exercise than a problem to be solved. But in real-life situations, such as determining what per cent of the automobiles driving past the school are violating the speed limit, computation is only one of the steps in a sequence of problem-solving stages.

These stages have been identified and can be listed in student's words as:

--Getting to know the problem
--Deciding what to do
--Doing it
--Thinking about what you have done

Getting to know the problem is in many cases being able to restate and recognize the problem in terms of the student's own experiences. Once the problem is recognized, the task is to put into motion a strategy to resolve it. For most students, this "deciding what to do" stage is to use computation to solve the problem. There are, however, other problem-solving strategies that students may need to solve real world problems. Using guesses, using models, using graphs, using equations, using tables, using diagrams, and using resources, are some of these other strategies.

The "doing" stage of the problem-solving is putting into action a single strategy or combinations of strategies. Since the "doing" stage of the problem-solving process generally involves computation, it's at this point that the calculator can make it's impact. If a student comes to the problem situation feeling mathematically insecure because of limited computational skills, this unnecessary intimidation diminishes chances of solving the problem and greatly affects overall problem-solving behavior. It is also the calculator that makes the "thinking about what you have done" stage of problem-solving possible for many students. With the calculator, students are more willing to recheck and verify their solution and at times even extend the problem to find out more about the situation.

For example, students used hand-held calculators to compare the populations of North Dakota and Rhode Island. Knowing that North Dakota has an estimated population of 642,000 and an area of 183,065 square kilometers, some students computed the population density of North Dakota and found it is about 3.5 people per square kilometer. They also computed the population density of Rhode Island and found it is about 302 people per square kilometer. Students got a better understanding for what it's like to live in North Dakota and Rhode Island when they used their calculator and extended the problem to find that about 55,000,000 more people would have to enter North Dakota for that state to be as densely populated as Rhode Island.

Junior high mathematics programs should offer students the opportunity to improve their problem-solving behavior. This means there should be a greater emphasis on problem-solving in the junior high grades. The hand-held calculator will play an essential role in this shifting emphasis.

THE SENIOR HIGH LEVEL

Roles the hand-held calculator will play in the high schools of the future should reflect the varying needs and aspirations of the students. Students will be considered in three categories: (1) those pursuing and reviewing the most fundamental principles of mathematics needed to be knowledgeable consumers and citizens; (2) those seeking to extend their understanding of the quantitative and spatial relationships needed for decision-making; (3) those who will continue their study of mathematics in post-secondary education.

Students currently in the first category are enrolled in remedial mathematics classes in which computational proficiency is sought, but seldom attained. In the future, the goal of proficiency with paper-and-pencil methods can be abandoned. Student dependence on the calculator is preferable to the alternative, viz., reliance on an imperfect mastery of the computation facts and algorisms. The cost of a calculator itself and its upkeep is a small price to pay for the power gained. Very likely the calculator itself will be no more cumbersome than a pocket watch. The mathematics curriculum for these students must, of course, be drastically changed. Students must master the skills in using the calculator with the basic operations--addition, subtraction, multiplication, and division. Complementing instruction in the mechanics of using the calculator is instruction in certain recording skills which are needed in more complex, multi-step problems. Although these skills are prerequisites for further study and for life itself, they should not be prominent in the curriculum. Some students will already have the requisite skills and others will acquire them in just a few hours of instruction.

With computation skills enhanced by the calculator, students are free to explore mathematical activities designed to build their problem-solving competencies. Activites should center on mathematical understanding as well as practical applications. A few selected examples illustrate use of the calculator for these students:

> Project: Compare the cost of
> traveling alone from Chicago
> to New Orleans by car, by bus,
> by train, and by airplane.

Data gathering is left to the students or a team of students. The calculator is used to add distances, determine fuel consumption and costs, and so on.

Another example shows how the students might use a simple, given algorism for the calculator to solve per cent problems.

30% of 45 is _____

45 $\boxed{\times}$ 30 $\boxed{\%}$

The display shows 13.5

30% of 45 is 13.5

Complete these:

30% of 150 is _____

25% of 70 is _____

Another type of practical application suitable for students with a calculator is illustrated in this example of algorismic presentation.

To find fuel consumption rate,

Enter: Kilometers traveled

Press: $\boxed{\div}$

Enter: Liters of gasoline used

Press: $\boxed{=}$

Display: Km per liter

Calculate the km/liter for these:

230 km, 21 liters __________

315 km, 34 liters __________

A practical application typical of many confronting householders today is illustrated in this example.

Project: Select from a mail-order catalog 5 items you would like to buy. Complete the order blank including the postage and sales tax.

Another example of practical mathematics made functional by the advent of the hand-held calculator is the traditional better buy problem.

Two 60-watt bulbs are available. One is rated at 600 hours and costs 39¢. The other is rated at 800 hours and costs 49¢. Which is the better buy?

Helping the student become a wiser consumer is one of the greatest services mathematics can provide the student.

Instruction for the students in the first category is probably not best organized in traditional full-year courses. Instead it may better be organized in short courses or units, none exceeding one semester duration. A student could take several such courses during the high school years.

Students currently in the second category, if they are in mathematics, are usually in courses in general or consumer mathematics. Some are misplaced in the analysis sequence; many take the minimum courses. A relevent curriculum and materials is not available for the students in many present-day schools. A wide variety of materials are needed to meet the various interests of students. These might be called mathematics units for general education. Many of the students will continue their education in colleges or vocational-technical schools. The instructional activities may best be organized in a variety of short courses, none more than one semester long. The needs of future mechanics, artists, plumbers, school teachers, and retailers are diverse but can be accommodated in specialized courses.

Uses of the hand-held calculator are exemplified in these illustrations:

Foundations of Mathematics Short Course

Solve: $23x + 115 = 966$

The foundations students with their calculators may use a guess-and-test approach to solve the equation. However as they encounter other equations of the same form, perhaps with non-integer solutions, the need for more sophisticated methods becomes apparent. The calculator provides the students with the power to explore their hunches and to test their conjectures.

Geometry Short Course
Use the distance formula to
find the length (to the
nearest hundredth) of the
segment joining points
(3, 7.3) and (9.7, 20).

The calculator with a square root key allows the student to perform all the calculations rather easily and to focus attention on the geometric relationships.

Statistics Short Course

Measure the length of the twenty pencils in the box. Calculate the mean and standard deviation of the lengths.
Materials needed: 20 pencils of assorted lengths.

The calculator allows students to perform experiments, generate their own data, and operate in the real world. If the measures yield awkward data, the student is undaunted; the calculator can handle it. The student has more time to concentrate on the statistical concepts.

Short Course in Finance

Find how many years you must wait to have $1,000 if you invest $500 under these conditions:

a) 6%, compounded quarterly
b) 9%, compounded quarterly
c) 12%, compounded quarterly

The calculator allows the student to experience firsthand the effect of the rate of interest on the doubling time. The student has a tactile and visual experience as well as an intellectual one. Graphs and tables become summaries of the experience, not beginning points. Tedious calculations with logarithms are no longer necessary.

Short Course in Finance

You buy a car for $3500 and agree to pay $150 each month. You are charged interest at the annual rate of 12% on the balance remaining each month. Prepare a table showing the balance remaining for each month. What is the total cost of the car?

The problem above is quite manageable with a hand-held calculator and almost impossible with pencil-and-paper methods. Because of its practical relevance to high school age students, the problem could also be included in a course in Consumerism. Here's another simple example:

Short Course in Consumerism

You can buy potatoes at two prices:
a) 3 kg for 89¢ or
b) 8 kg for $2.09.
Because it takes longer to use, 10% of the larger quantity is lost to spoilage. Which is the better buy?

The use of formulas is a part of every person's life, whether it be preparing a tax form or reading an actuary's handbook. A simple example is given below:

Short Course in Mechanics

Use the formulas:
Ampers = Volts/Resistance and
Watts = Volts x Ampers
to calculate the amperage and the resistance for these appliances using 110 volts.

4-slice toaster	1600 watts
Light bulb	60 watts
Heating pad	72 watts
Coffee maker	600 watts
Electric iron	1200 watts

The student using a calculator can perform all calculations quickly and accurately. Fear of mathematics, when mathematics is synonymous with computation, is removed and the student is free to work on more important things. This change in focus is possible whether the course be one in mathematics itself or a mathematics-related unit in social studies, business or science.

The third category of student, one who will continue studying mathematics after graduating from high school, is better accommodated by the present-day mathematics programs than students in the first two categories. These students typically take two courses in algebra, one in geometry, plus a fourth-year course. Although the calculus-preparation sequence will undoubtedly undergo considerable change in the coming years, the remainder of this paper will consider the effect of the hand-held calculator on existing courses.

In the first course in algebra, students typically study systems of linear equations in carefully controlled settings. The equations are written in standard form with two variables having integer coefficients. The solutions are pairs of integers or simple rational numbers. The most difficult systems of equations the student encounters include fraction coefficients; some may experience very simple equations in three variables. Although students may reach the level of generalization in which they solve systems of equations with literal coefficients, it is very difficult for them to solve many equations which they write for themselves or which arise in real-world situations.

The advent of the hand-held calculator allows instruction to focus on some aspects of the topic which have previously been unattainable for many students. No longer is the awkwardness of solving and checking systems of linear equations with rational coefficients an insurmountable barrier. At long last students may encounter practical applications of their study. Having reached the stage of generalization, they may now, in fact, solve any system of two equations in two variables. No longer need the solutions be restricted to simple integers. Large numbers and decimals may be solutions from the beginning. Freed from computational restrictions, teachers and students may indeed focus on the concepts. The principles used in solving systems of linear equations remain the same but now the principles are functional. Simple guessing or graphing approaches may still be used to provide an overview to the topic but they will soon prove to be inadequate in solving typical problems. For example, early in the study, the student may be asked to solve the system:

$$\begin{aligned} 2.7x - 1.03y &= 28.418 \\ 31.4x - 11y &= 451.24 \end{aligned}$$

To find the solution $(57.6, 123.4)$, the student must understand the principles of algebra and operations with rational numbers, but computation hurdles are removed. This equivalent system is easily obtained with the hand-held calculator:

$$\begin{aligned} 29.7x - 11.33y &= 312.598 \\ 32.342x - 11.33y &= 464.7772 \end{aligned}$$

The division needed to solve $2.642x = 152.1792$ is now easily accomplished in a matter of seconds. With pencil-and-paper methods, the division would be horrendous. Whether the solution $x = 57.6$ is correct cannot be verified by inspection. Judgment must await the ultimate test; does replacement of the number pair solution in the original equations produce true statements? The check is no longer a trivial mental exercise, often ignored, but becomes an important final step in the process. If an error is detected, the student has time to correct it. A common type of error, computation mistakes, is practically eliminated.

Why do graphing problems avoid the skills needed to solve many practical problems? Students seldom must make decisions regarding the scale to use but instead they face only simple equations such as $y = 3x - 5$ where the y-intercept is an integer close to zero and the slope is at worst a simple rational number. Until the calculator was available, an equation such as $y = 1.07x + 54.6$ was generally excluded from experiences in graphing because of the computational difficulties it posed. To generate number pairs for its graph would tax even the most able student. With a hand-held calculator, however, any algebra student can quickly generate several number pairs and verify the linearity of their corresponding points. Freed from computational drudgery, students can graph any line. Other, more important skills will become the focus of the class. Even the graphs of higher degree polynomials become accessible to algebra students equipped with hand-held calculators.

Like their linear counterparts, graphs of quadratic functions and solutions of quadratic equations may be made more like real-world examples. Unlike when working with linear models, however, the student will find that a calculator equipped with a square root function is especially useful. Many inexpensive calculators come equipped with the square root key; nearly all moderately priced models do. One scientific model would meet the needs of a classroom. With hand-held calculators available, teachers and students should quickly move toward the quadratic formula as the method used in solving quadratic equations. Factoring techniques and the completing-the-square method should be used only as long as needed to develop the quadratic formula. Long an overused and archaic method, factoring quadratic expressions should be given little time in the algebra classroom.

Although the study of logarithmic functions could become an interesting unit with practical applications in an algebra course, with calculators available a different emphasis is needed than has commonly been the case. With scientific calculators available in the classroom, logarithms as computation aids are now outmoded. Computation needs are better met by the calculator. As a mathematical model, however, the lorarithmic function will be quite useful to the mathematics student.

Exponential functions are made much more understandable to the student with a calculator. Laws of exponents are easily taught inductively. Many experiences can be made available in a short period of time with computation done on a calculator. Practical applications which were avoided when students used tables or logarithms now are routinely within their grasp. An example illustrates this point.

Find the value of a \$500
investment after 10 years
when invested at $5\frac{3}{4}\%$
compounded quarterly.

Even the simple hand-held calculators can easily find

$$500(1 + \frac{.0575}{4})^{40}$$

Scientific models can find the answer efficiently. Even the pesky zero-exponent difficulty is more teachable.

In the second course in algebra computation has seldom been a problem. Solutions have often been left in computational form, sometimes using non-decimal notation such as π, e, and expressions using radicals, logarithms, exponents, and trigonometric symbols. Now, if it fits our purpose, with the aid of a calculator these expressions may easily be converted to standard decimal notation. The distinction between the irrational numbers and their decimal approximations could be a topic of study; in problem solving, however, there is no loss of utility when eight-digit approximations are used.

No longer must trigonometric expressions and applications be controlled to avoid computational difficulties. Simple calculators provide more than enough accuracy to be used with standard trigonometric tables. Scientific models generate the trigonometric functions of radians or degrees with the press of a button; no tables are needed. The student with a scientific calculator could verify the accuracy of commercial tables and investigate further the series used to generate them. By dividing the length of each side of a triangle by the sine of the angle opposite it, the student with a calculator may inductively be led to discover the Law of Sines. Without a calculator the computations would be too time-consuming to make the experience worthwhile.

The Law of Cosines becomes a more powerful and meaningful principle to the student when it can be applied to actual problem solving situations. No combination of numbers is too difficult to handle, even on the simple models.

The graphs of composite trigonometric functions become accessible to the student with a calculator. For example, $y = \sin x + 2 \cos x$ may be plotted, point-by-point, and its features examined. Period, amplitude, and frequency may be studied inductively through many experiences made available through the calculator.

The solving of polynomial equations beyond the second degree becomes feasible with iterative techniques and a calculator. The principles long used to locate zeros of a polynomial become functional as the student applies the principles to find the roots of equations. Direct or synthetic substitution may be used to evaluate polynomials quickly so that even graphs of polynomials with non-integral coefficients are within the reach of students using calculators. The effect of various parameters can be explored with the drudgery of computation removed.

Applications of conics would not have to be fabricated to fit a simple mathematical equation. Actual orbits of planets, cross sections of headlights or roadbeds, and paths of projectiles become a source of actual problems that students can solve.

In geometry courses, teaching may now include more inductive and empirical methods. Consider this example:

> On scratch paper, draw 10 triangles in a variety of sizes and shapes. For each triangle, (1) join the midpoints of two sides and (2) divide the length of this segment by the length of the third side. Write a conjecture based on this experience.

The empirical data may lead to a hypothesis which is later proved deductively. Another example illustrates the same point.

> Draw eight circles of different sizes. For each circle, (1) draw two chords which intersect inside the circle and (2) calculate the product of the lengths of the two parts of each chord. Write a conjecture based on this experience.

The calculator provides a valuable aid to the study of many topics in analytic geometry. Use of ratio and proportion is routine on a calculator and the student and teacher are freed to concentrate on the concepts. When area and volume computations are handled with a calculator, greater attention can be given to units of measure and to relationships among the figures. The distance formula in either two- or three-dimensional spaces becomes more functional with the aid of even a simple hand-held calculator. Applications of the Pythogorean Theorem are routinely handled.

In courses where determinants are calculated, the hand-held calculator will be a valuable aid. Probably only when programmable calculators are available would much time be given to evaluating determinants of the third and higher order. Entries in matrices could better fit real-world situations; computational difficulty no longer would be the controlling factor.

Contemporary courses or units in probability and statistics suffer without the availability of calculators. Problems are often controlled or contrived to keep computational difficulties within the grasp of students using pencil-and-paper methods. Consequently, students have little confrontation with real-world problems. With calculators, however, students can successfully find means, variation, and correlations of sets of data of revelance to them. The principles of the course become the focal point.

As in the mathematics courses, calculators should be a common tool in many classrooms. The science teacher no longer need bemoan the lack of computational skill in science students. With calculators available to handle the computation, the science class is better able to deal with the concepts of the course. Science teachers and mathematics teachers should be better able to correlate essential aspects of their courses when computation concerns are lessened.

Students in business and social studies should have free access to their own or the school's calculators. Quantitative aspects of the courses should gain greater prominence with computation hurdles lowered. Examples from the real world may be used, bringing greater relevance to the classroom and increasing student motivation.

Perhaps special mention should be made of the role of the calculator in testing. Since the calculator will become an integral part of instruction and an indispensable tool of the students, their use should be allowed during tests. The only exception would be when the subject of the test is computation itself. To not allow calculators to be used on tests in future would be as senseless as not allowing the use of pencil and scratch paper today.

The use of calculators on tests has implications for the test writer. Care must be taken to be sure that students use the intended technique. Consider how a student armed with a calculator might solve this multiple-choice item:

What is the solution set of

$$2.53x + 1.34y = 15.017$$

$$1.16x - 3.97y = 1.384$$

A) $\{(7.2, -2.4)\}$
B) $\{(3.2, 5.4)\}$
C) $\{(-4.8, 20.3)\}$
* D) $\{(5.3, 1.2)\}$

The student could check each of the four responses in the two equations until the correct solution is found. So the astute student could correctly answer the question without knowing how to solve a system of linear equations. This slightly different item is better suited to the class with calculators:

What is the value of x in the solution set of

$$2.53x + 1.34y = 15.017$$

$$1.16x - 3.97y = 1.384$$

A) 7.2
B) 3.1
C) -4.8
* D) 5.3

Of course the concept could easily be tested directly with a short-answer item:

What is the solution set of
$2.53x + 1.34y = 15.017$
$1.16x - 3.97y = 1.384$

With computation needs met by the calculator, tests may be more comprehensive than previously and focus on the principal concepts.

SUMMARY

1. The calculator is inevitable.
    a. Cost is below $10 for basic machines.
    b. The per pupil cost is less than the present textbook costs.
    c. One out of five adults already has a calculator.
    d. Within a few years nearly every adult will have a calculator.
    e. Every student today has a calculator in his future.

2. The calculator will affect the present curriculum.
    a. The present curriculum is based on pencil-and-paper computation. A lesson that takes fifty minutes with pencil and paper may take less than ten minutes with a calculator.
    b. The existence of the calculator affects the computation "need to know" level for all students. We no longer need to spend time to try to get all students to divide a five-digit number by a three-digit number for example.

c. The calculator and the metric system makes the decimal fractions more important than the common fractions.

d. Many concepts can be better taught using a calculator. Examples include estimation, mental arithmetic, large and small numbers, and problem solving.

3. The calculator is a great equalizer. Students who could not learn mathematics because they could not compute can now learn when the development is supported with a calculator.

CONCLUSIONS

1. Materials
    a. The existence of the calculator and its implications for the curriculum require the development of new software at all levels.
    b. New course offerings that better meet the needs of various subsets of the student population are possible because of the existence of the calculator. For example the dilemma presented by the usual general mathematics course may be solved when all students can use the calculator.
    c. New materials need to be created that focus on the development of general objectives. It is not enough to determine

if the calculator is good or bad. Rather we need to find how to use it to teach concepts such as decimals, estimation, or problem solving.

2. The teacher
    a. All teachers should be encouraged to experiment with the use of the calculator in the classroom. Experience will help the teacher make wise and proper decisions in the future.
    b. Teacher educators should be encouraged to use calculators in pre-service classes for teachers.
    c. Inservice courses in the use of calculators to teach mathematics need to be developed. Unless the teacher is ready, materials will not be effective.
3. Evaluation
    a. Experimentation in the design and development of evaluation materials using the calculator should be included in present evaluation efforts.
    b. Present paper-and-pencil tests need to be modified to recognize changing curriculum objectives.

c. Evaluation instruments need to be developed to determine the effectiveness of programs using the calculator.

d. Evaluation of calculator use itself will also require new instrumentation.

4. Parents

Any general change of the curriculum should involve the public. As the public become general users of the calculator and as they recognize that the calculator is being used to teach the basic objectives better, support of the use of the calculator in schools can be expected. If this information is not cummunicated, one can expect considerable resistance to the change the calculator implies.

5. Research

The calculator itself is neither good nor bad. As with any other tool we need to find out when and how it can be used effectively. Studies designed to assess the effectiveness of teaching using the calculator with textbook materials designed for pencil-and-paper skills are likely to be of little value. On the other hand, experimental studies with specific objectives and with specific software need to be carried out at all levels. These experiments will give guidance, not to whether calculators are good or bad, but to how they can and should be used.

APPENDIX D

HAND-HELD CALCULATORS AND POTENTIAL REDESIGN OF THE SCHOOL MATHEMATICS CURRICULUM

by

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In recent years a number of new techniques and devices to help with teaching have come along, have enjoyed a brief period of enormous and somewhat uncritical enthusiasm, and then have been added in a limited way to our arsenal of available pedagogy. These include, to name just a few, programed instruction, films, modular scheduling, and computer assisted instruction. It is not entirely unfair to recall that some of these were announced initially by their devotees as the solutions to all pedagogic problems. In their first wave of excitement, proponents included many glorious things that they could do with these new techniques, and in particular how these could replace or unburden the teacher in one or another of her many functions. This human enthusiasm is perfectly natural, and I do not wish to belittle it. It is, however, unfortunate that in many cases the mathematicians looking at these new pedagogic techniques reacted in exactly the same uncritical spirit, that is, with either missionary enthusiasm or uncompromising disdain, rather than taking a deeper look at the problem. For example, when the wonderful new ability to make films for mathematical purposes became easily available -- along with suitable funding --

many mathematicians joyously jumped in. They thought of all sorts of beautiful phenomena they could show on film and they had a wonderful time. Unfortunately, and it is very much easier to say this in retrospect than it would have been to say it at the time, mathematicians did not choose to ask the probably more interesting question: What are the most difficult pedagogic problems we have in mathematics education and how can films help us overcome these? If we had stopped to consider the problem from this angle, we might have made films on conditional probability or curvilinear motion, or even a brief segment of animation to help with that nasty problem of the volume of intersection of three mutually perpendicular cylinders, rather than making films on area and limits. The latter are perfectly fine topics, but it is not clear that there is much marginal gain for film over a blackboard. In hindsight, we might have heavily emphasized three dimensional and motion problems in our films -- that's where the blackboard really can't compete.

We find ourselves at this time with the great opportunity opened up by another pedagogical technique, another device to help us with our teaching, namely the hand-held calculator. Once again it is perfectly natural to discuss all the glorious things that you might be able to do, and all the ways in which some functions of the teacher might be replaced. Not unnaturally, given the previous paragraph, I should like to urge consideration of the other side of the coin. What are some of the most difficult problems we have in teaching school mathematics with which the calculator might help? To keep you from reaching the immediate prediction that this might be the empty set, let me give an example. I am told that we often have great problems in teaching the notion of a function. It is difficult for the students to get a clear hold of the idea that what matters (in the simplest case) is that when a number goes in, a single number comes out, in fact the same number every time. We try to get the students to realize that any way of describing how the number that comes out is related to the number that goes in is perfectly fair game, and leads to the same function. For example, we may describe the function by a table, or by a graph, or as a story in words, or by a formula, or by an arrow diagram, or by various other things that people have tried. I think that one more useful device in this collection will turn out to be the hand-held calculator: A function can also be described by a fixed routine which you follow on the hand-held calculator, a determinate series of buttons to push and things to do at the end of which the number you want comes out. The calculator approach may also help to prevent the misconception that a function is a formula.

This may seem like a relatively small improvement, although I believe strongly that the physically active nature of this approach will make it the best approach to functions for some students. However, in a particular piece of the problem, namely that of inverse functions, I can see the possibility of a real improvement for everybody. My impression is that our analytic ways of describing inverse functions have often gotten bogged down in notation and have been difficult for many students. After all, the statement $g(f(x)) = x$ is a little much for the student who is still very uneasy about $f(x)$ itself. Reflecting the graph across the line $y = x$ is also difficult for those students who are struggling with variables and what they mean at the same time they are fighting the $f(x)$ notation. The hand-held calculator provides an opportunity for the student to understand inverse functions by actually experiencing the process. The original function starts out with a number and gives us a second number. The inverse function takes the second number and gives back the original. You can, of course, see this immediately in the use of x^2 and $\sqrt{x}$ keys, of the exponential and logarithmic keys, or the keys for various direct and inverse trigonometric functions. But there are many fancier but revealing things you can do. The inverse to

$$y = 8x^3 - 36x^2 + 54x - 27$$

is

$$x = \frac{1}{2}\,(y^{1/3}+3).$$

The inverse to

$$y = \sin \frac{x}{1+x}$$

is

$$x = \frac{\text{arc sin } y}{1 - \text{arc sin } y}\,.$$

My hunch is that if students experience the transformation of numbers by such pairs of programs on the hand-held calculator and see how it all comes out they will get a good understanding of inverse functions, one that is hard to obtain any other way.

Besides the areas associated with the understanding of functions there are many others which immediately come to mind in which the hand-held calculator might lead to real pedagogic advantages. To name a few, we might now be better able to teach iteration methods for solving simultaneous linear equations, or, later on, some more general nonlinear equations. In the learning of probability, the hand-held calculator might greatly increase the variety of experiments which can be the source of data to be studied and used as illustrations. In practical statistics, data analytic computations might become more accessible. So might linear programming. This list is not meant to convey any lengthy consideration of the subject. It is simply to confirm the notion that there are probably many mathematical topics with real pedagogic difficulties where the hand-held calculator might help.

If further reflection indicates that a rich variety of such opportunities actually exists -- as I believe it does -- this opens up the possibility to rethink the curriculum in a much broader sense. I believe that the curriculum should be based in a fundamental way on two partial orderings, one of which is essentially supplied by the discipline and the other by society. The partial orderings I have in mind are those of prerequisites and of importance. It is in many cases true that one mathematical topic really has to precede another, and the design of the curriculum must take this into account. It is up to the experts on the mathematical and pedagogic sides of the house to make clear the existence of such prerequisities, although it is worth pointing out that these experts sometimes change their minds. For example, we had always assumed that work with fractions must precede any work on probability. When we finally realized that probability is perhaps the best available motivation for work with fractions, we began to experiment seriously with the opposite order. This does not, however, detract from the point: there is among mathematical topics, or clusters of topics, a partial ordering of prerequisites. Within a cluster, different orders may indeed be practical. The second partial ordering, as I have indicated, is one of importance. There are some mathematical ideas, topics, techniques which are more nearly essential than others for the population as a whole. For example, to take a simple, but not altogether untimely, case, I would maintain that probability is more important for the population at large than the division of polynomials. Unfortunately not very long ago we taught division of polynomials in the 9th grade and did not teach probability until much later if at all. Division of polynomials is probably needed for the first time for partial fraction expansions in second year calculus (that's the prerequisite side of the argument) and not very important in its own right; probability is probably needed for the first time in the elementary school and is enormously important for everybody. Of course, these were probably not common opinions when the traditional mathematics curriculum became solidified. But societal needs, and societal views of the mathematical sciences, have a way of changing, and they influence the mathematics curriculum through this second partial ordering of importance.

If my views on the pedagogic possibilities of the hand-held calculator are realistic, then hand-held calculators can have a major effect on both the partial orderings we have been discussing. The ability to handle numbers and functions and algorithms in new ways will loosen and even alter some of the prerequisites we have always believed. We will be able to use the ability to work with division of numbers and with trigonometric functions and with exponents and logarithms as motivation for a deeper study of these topics rather than having to insist that the study must precede any practical use of the functions and techniques. Not that we will necessarily always want to do this. However, the possibility of inverting the order of experiences gives us a flexibility to reconsider much of the curriculum. Similarly, the hand-held calculator may allow us to take up a number of topics early, topics which are of great importance to various populations but which we have not been able to approach previously. These are, for example, some of the mathematical topics which we mentioned previously and with which we have had pedagogic difficulty in the past. To mention just one, if some rudimentary ways of looking at data are more important to everybody than factoring, then we can with the aid of the hand-held calculator perhaps do data analysis earlier and more successfully, and demote factoring to a less prominent position.

There is no obvious single way in which the community which plans for education in the mathematical sciences must organize itself to take advantage of the new opportunities deriving from the easy availability of hand-held calculators. Let me however mention just one possibility. It might be sensible to have a conference, in the general pattern and scale of the Cambridge conferences during the previous decade, to reconsider the school curriculum with this new point of view in mind. I am not saying that the same mix of backgrounds of the participants in the various Cambridge conferences is necessarily optimal; in fact, we will need a greater proportion of people knowledgeable in the many aspects of mathematics education that the first Cambridge conference included. However, a month in some relatively secluded spot, with day- and night-long churning and bubbling of ideas, is not a bad way to proceed although admittedly nontrivial to organize. Out of such a conference could come a blueprint to help guide the pattern of the evolution of education in the mathematical sciences for the near future.

APPENDIX E

SOME SUGGESTIONS FOR NEEDED RESEARCH ON THE ROLE OF THE HAND-HELD ELECTRONIC CALCULATOR IN RELATION TO SCHOOL MATHEMATICS CURRICULA

A Position Paper

Prepared by

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What should be the nature of school mathematics curricula vis-à-vis the calculator[1] and its instructional potential?

There certainly is no simple, single answer to that question. And it is just as certain that ultimate answers to the question must come in no small measure from research, viewed broadly in the sense that "Research is controlled inquiry" (Suydam & Weaver, 1975a). Before considering particular aspects of such research, I wish to clarify several things.

What Is a "Curriculum?"

I have adopted Kieslar & Shulman's (1966) characterization that "a curriculum refers to the organization and sequence of a subject matter in which statements about that subject, methods of teaching, and the activities of the learner are intricately interrelated to form a single entity [p.190]." This is the sense in which Romberg & DeVault (1969) used the term curriculum in their exposition of a model for mathematical curriculum research, as portrayed by Figure 1:

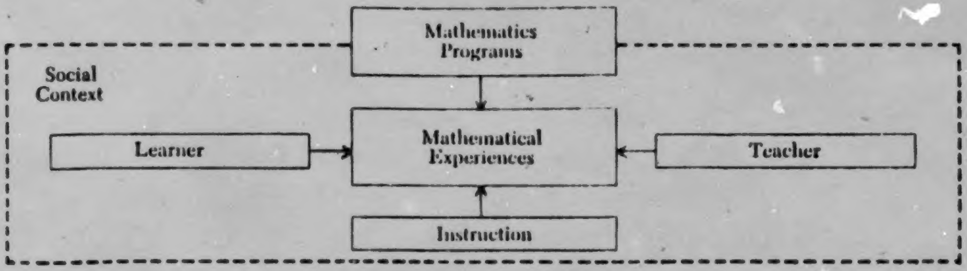

Figure 1. Mathematical curriculum research model (Romberg & DeVault, p. 96)

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[1] Throughout this position paper I have used the single word calculator to mean particularly the "mini" or "hand" or "hand-held" or "pocket" electronic calculator that is of express interest in this project.

This interpretation is chosen deliberately because it emphasizes both the scope and interrelatedness of factors that need to be considered in connection with research on school mathematics, which includes research pertaining to the use of calculators. A similar stress upon scope and relatedness of factors is evident in DeVault & Weaver's (1970) framework for discussing major issues (Figure 2) and principal forces (Figure 3) associated with elementary-school mathematics, which can be extrapolated readily and validly to higher levels of pre-college mathematics.

I believe that the expression "a school mathematics curriculum" commonly is interpreted, even in the Kieslar & Shulman (1966) sense, to embrace one or more grades. However, I find it convenient--and not at all improper--to apply the term curriculum more flexibly to include a "single entity" that is of much shorter duration: e.g., a curriculum associated with the addition of non-negative rational numbers expressed in common-fraction form; or a curriculum that is implicit in a particular 20-minute "lesson" with second-grade pupils.

It is with this more flexible interpretation clearly in mind that I chose to phrase my overriding question as,

"What should be the nature of school mathematics curricula vis-à-vis the calculator and its instructional potential?"

This is no longer a question. Rather, it is a host of questions!

A Potential Impediment to Productive Research

"The only thing we have to fear is fear itself" (Roosevelt, 1933).

The greatest thing we have to fear today about the calculator vis-à-vis school mathematics curricula is the degree of fear that already exists about the calculator vis-à-vis school mathematics curricula. If you will forgive a mathematical pun: The nontrivial degree of this fear is in no way simply imaginary, and can be a very real threat to productive research.

There is clear evidence of this fear in the very nature of many of the uses suggested for calculators in connection with school mathematics instruction

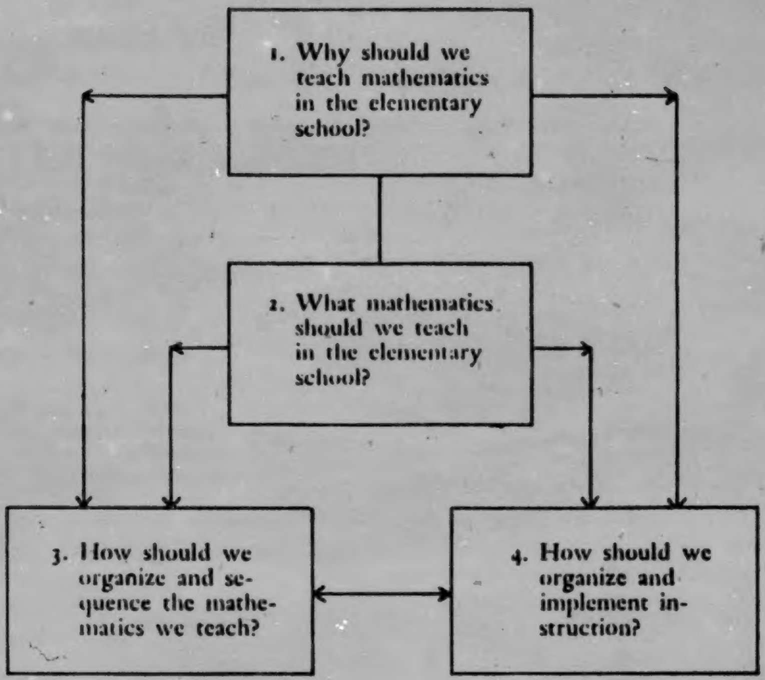

1. Why should we teach mathematics in the elementary school?
2. What mathematics should we teach in the elementary school?
3. How should we organize and sequence the mathematics we teach?
4. How should we organize and implement instruction?

Figure 2. Major issues (DeVault & Weaver, p. 95)

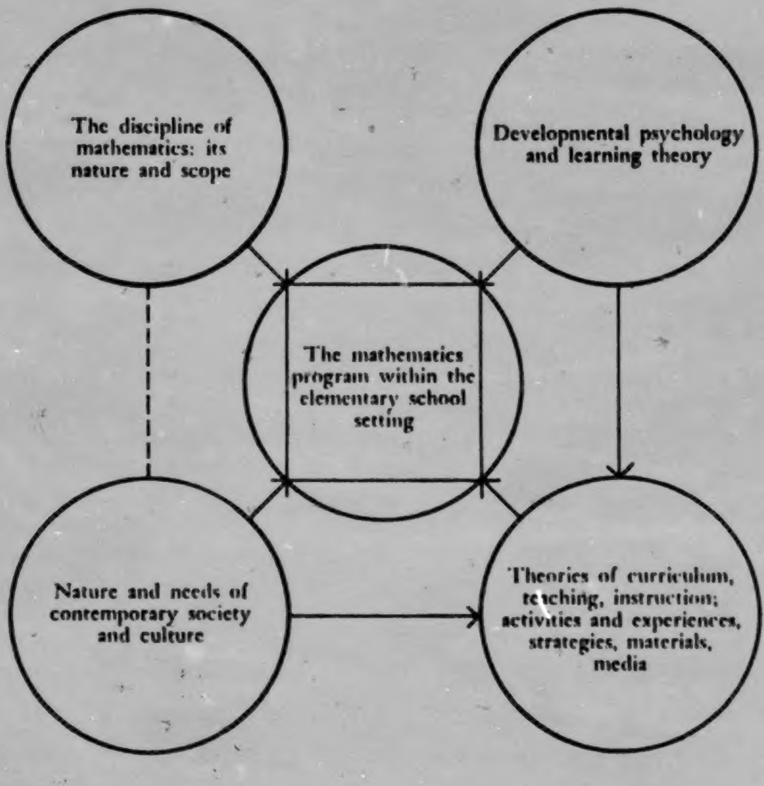

The discipline of mathematics: its nature and scope
Developmental psychology and learning theory
The mathematics program within the elementary school setting
Nature and needs of contemporary society and culture
Theories of curriculum, teaching, instruction; activities and experiences, strategies, materials, media

Figure 3. Principal forces (DeVault & Weaver, p. 96)

and in the caveats accompanying those suggestions. Let me illustrate.

Consider first the following "position statement" developed by the Instructional Affairs Committee (AIC) of the National Council of Teachers of Mathematics (NCTM) and adopted by its Board of Directors in September 1974:

> With the decrease in cost of the minicalculator, its accessibility to students at all levels is increasing rapidly. Mathematics teachers should recognize the potential contribution of this calculator as a valuable instructional aid. In the classroom, the minicalculator should be used in imaginative ways to reinforce learning and to motivate the learner as he becomes proficient in mathematics [underlining added]. (See "NCTM and the minicalculator" in the List of References.)

This innocuous statement--strikingly unimaginative in contrast to its emphasis upon the imaginative--admittedly "is only a first step in looking at a new topic [?] in mathematics education." (Additional suggestions from NCTM's IAC are to be published in the January 1976 issues of the Arithmetic Teacher and the Mathematics Teacher.) But I find it more than a bit disturbing to see such a milquetoasty "position statement" welcomed by many as gospel. Buckwalter (1975), for instance, mentioned that "authoritative word came down from the National Council of Teachers of Mathematics. With remarkable foresight, it pronounced the calculators a valuable instructional tool and predicted they're here to stay [p. 13]."

More recently the President of the NCTM (Gibb, 1975) has expressed the following point of view:

> How can the calculator be used as an instructional aid to enhance learning beyond what we might otherwise expect? . . . One guide for us all . . . is that we not use the calculator until our students have developed a concept of number, a system of naming numbers, and an understanding of the meaning and processes of the basic operations--that is, until our students understand what the calculator is doing for them. . . . Creative use of minicalculators after students' mathematical understandings have been abstracted . . . can establish it as a valuable asset among the in-

structional devices already in today's mathematics classrooms.

As I scan the relevant professional and technical literature, I find this point of view to be taking on the characteristic of a reverberation. Judd (1975), for example, cautions that

> Students must have a good background in manipulative math experiences before they can understand the inputs and outputs of the calculator. So it behooves us all to be very careful about putting calculators in the hands of children who do not yet possess a concept of numbers and their relationship to the real world. . . . Don't, in short, put a calculator in the hands of a student before he can use numbers to describe actual events, or before he understands the nature of the processes basic to arithmetic. Only after the students understand the meaning of the functions they are performing should they be given a magic box to carry them to completion [p. 48].

Grosswirth (1975) believes that "Unless their use is carefully monitored by teachers and parents, HHCS [hand-held calculators] can become a mathematical crutch on which poor or lazy students lean, to the detriment of their intellectual and mathematical development [p. 95]."

I sense that apprehensions are most acute at the elementary-school level and become progressively less acute at the middle- and secondary-school levels. As Grosswirth (1975) has indicated, "Philosophical questions introduced by HHCS are of minor significance at the college level [p. 90]."

To many persons the calculator threatens to violate certain tenets regarding school mathematics learning and instruction--tenets that are adhered to more tenaciously than I might have expected. Suggestions for calculator uses are made within the constraints of those tenets (allowing for a comfortable margin of safety), and any research that might be implicit in such suggestions would be similarly constrained.

Some other persons, however, appear to be willing--possibly even anxious--to suggest calculator uses that may challenge certain of our cherished tenets.

A striking example of this is found in the recently released report (1975) of the National Advisory Committee on Mathematical Education (NACOME) of the Conference Board of the Mathematical Sciences (CBMS). In the section of the report that deals expressly with calculators we read:

> while students will quickly discover decimals as they experiment with calculators, they will also encounter concepts and operations involving negative integers, exponents, square roots, scientific notation and large numbers -- all commonly topics of junior high school instruction. These ideas will then be unavoidable topics of elementary school instruction. For instance, students may discover from the calculator that the product of two negative numbers is a positive number and *computational facility with integers* (using the calculator) *will precede, rather than follow,* the *careful conceptual development* of these ideas [p. 41, underlining added].

This last sentence obviously is in marked conflict with the more commonly held, more conservative position adopted by the NCTM and expressed by its President and by numerous other persons--a position characterized by Etlinger (1974) as "the *pure-pedagogical* view" in which "the calculator must not be used to replace learning, but rather to facilitate learning" and in which "The electronic calculator will be a learning tool in the school programs as are the abacus . . ., Geoboards or Cuisenaire rods [p. 44]."

I can appreciate why some organizations and persons take the relatively cautious stand they do. But it would be a serious mistake if research on the nature of school mathematics curricula vis-à-vis the calculator were restricted to treating it simply as another (albeit powerful) instructional device, tool or aid, and if that research failed to be influenced to some nontrivial degree by NACOME's (1975) contention that "The challenge [of the calculator] to traditional instructional priorities [and practices] is clear and present [p. 41]."

Calculator-influenced School Mathematics Curricula

Figure 4 suggests that calculators may have different kinds or degrees of influence upon school mathematics curricula.

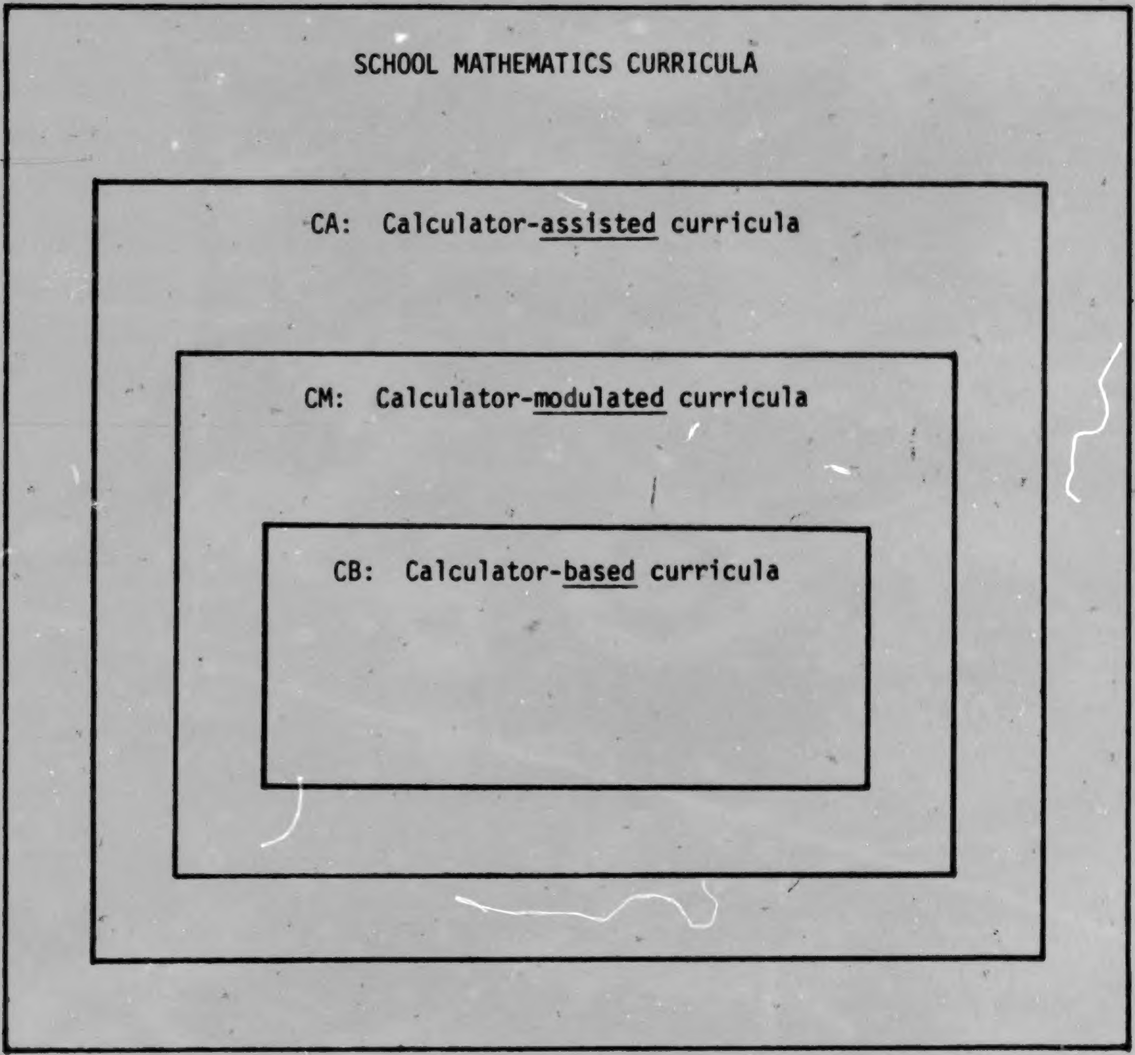

SCHOOL MATHEMATICS CURRICULA
CA: Calculator-assisted curricula
CM: Calculator-modulated curricula
CB: Calculator-based curricula

Figure 4. Calculator-influenced school mathematics curricula

There are some things for which use of a calculator is inappropriate, such as curricula associated with aspects of nonmetric geometry.

I have chosen the expression "calculator-*assisted*" (CA) to embrace all instances in which a calculator *is* used in some way in connection with a curriculum.

The intent of CA curricula, of course, is to facilitate instruction. This facilitation comes from use of the calculator itself and at times may be enhanced by some special feature of the calculator. Consider, for instance, the worksheets reproduced as pages 9-10 of this manuscript.

These are among the worksheets I used recently with two classes of third-grade pupils as part of our work with unary operators, in which we first took a "Guess and Test" approach to solving examples of the forms

$$\square \xrightarrow{+a} b$$

$$\text{and } \square \xrightarrow{-a} b$$

where a and b were whole numbers of the magnitudes illustrated. The use of calculators clearly facilitated this work. Furthermore, the facilitation is enhanced when using calculators with an "automatic constant" feature, in which the operator "+a" or "-a" for a particular example needs to be entered into the calculator via the keyboard only for the first of the guess-and-test trials. This CA approach led to the use of the worksheet reproduced as page 11 of this manuscript, where pupils developed an inverse-operator approach as a more efficient way of solving examples. In effect, pupils used the following equivalence relations involving unary operators:

$$\square \xrightarrow{+a} b \leftrightarrow b \xrightarrow{-a} \square$$

$$\text{and } \square \xrightarrow{-a} b \leftrightarrow b \xrightarrow{+a} \square.$$

Again, the approach was CA: calculators were used to solve an example after transforming it to an equivalent inverse-operator form; then calculators were used to verify the solution for the example as given originally. In all of this work the use of calculators was not necessary; but the CA mode of instruction clearly was facilitating. Also, the calculator's "automatic constant" feature definitely enhances that facilitation for the guess-and-test procedure.

Name:

1 2

$$\boxed{} \xrightarrow{-519} 423$$

GUESS and TEST

900	381
1025	506

etc.

E

$\boxed{} \xrightarrow{+459} 1542$

GUESS and TEST

________ ________

________ ________

________ ________

________ ________

________ ________

________ ________

________ ________

________ ________

F

$\boxed{} \xrightarrow{-739} 467$

GUESS and TEST

________ ________

________ ________

________ ________

________ ________

________ ________

________ ________

________ ________

________ ________

A $\boxed{} \xrightarrow{+476}$ 871 * * * * * 871 $\xrightarrow{476} \boxed{}$

B $\boxed{} \xrightarrow{+783}$ 1306 * * * * * 1306 $\xrightarrow{783} \boxed{}$

C $\boxed{} \xrightarrow{+695}$ 1542 * * * * * 1542 $\xrightarrow{} \boxed{}$

D $\boxed{} \xrightarrow{-549}$ 413 * * * * * 413 $\xrightarrow{549} \boxed{}$

E $\boxed{} \xrightarrow{-278}$ 536 * * * * * 536 $\xrightarrow{278} \boxed{}$

F $\boxed{} \xrightarrow{-793}$ 367 * * * * * 367 $\xrightarrow{} \boxed{}$

G $\boxed{} \xrightarrow{+584}$ 1000 * * * * $\xrightarrow{} \boxed{}$

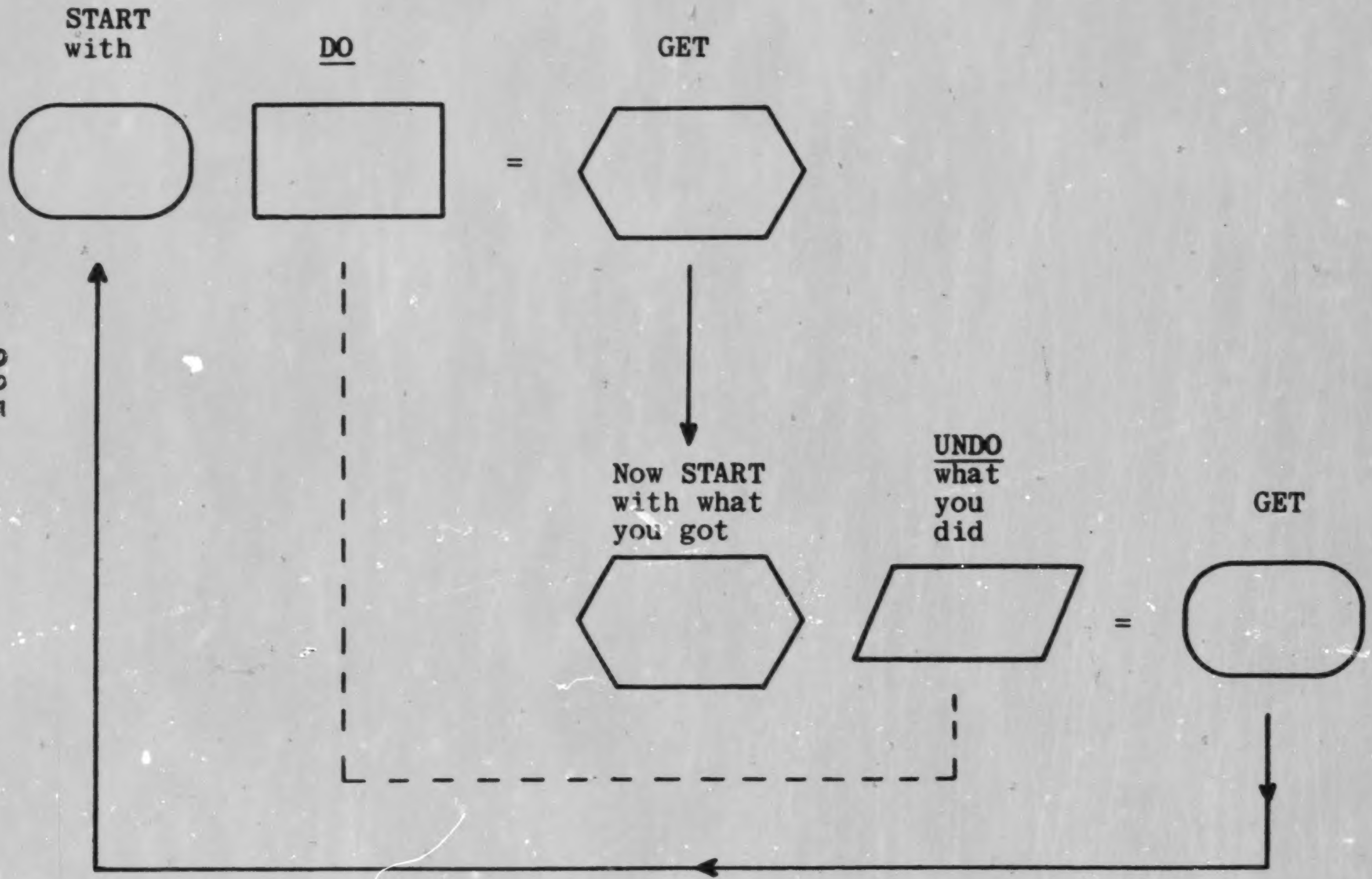
H $\boxed{} \xrightarrow{-485}$ 1000 * * * * $\xrightarrow{} \boxed{}$

Just in case you are curious, examples such as $\square \xrightarrow{+794} 385$ were used during the course of instruction![2]

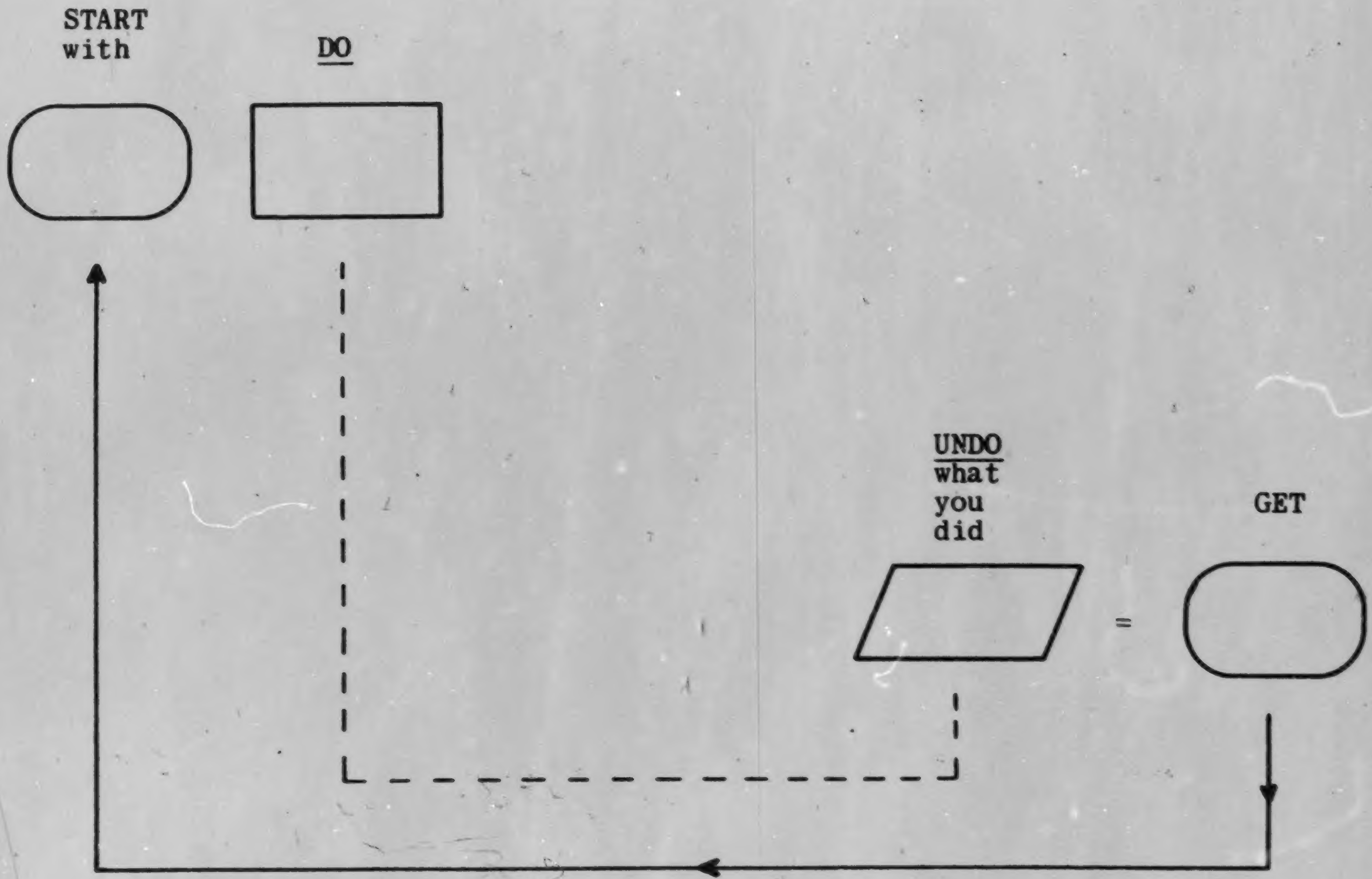
---

[2] These worksheets are drawn from my ongoing third year of activity pertaining to calculator-influenced explorations (Weaver, 1976), begun during the 1973-74 school year and continued during 1974-75 and into the current school year (1975-76). This present activity places a major emphasis upon unary operators, following some previous years' work with this approach as illustrated by the worksheets reproduced as pages 13 and 14 of this manuscript. The calculator-influenced explorations are a direct consequence of the investigations listed as "References: Supplement C."

The DOING-UNDOING Idea


START with
DO
=
GET
Now START with what you got
UNDO what you did
=
GET

The DOING-UNDOING Idea


START with
DO
UNDO what you did
=
GET

Calculator-modulated (CM) curricula. I shall use an illustration at the outset to explain my interpretation of CM curricula as a proper subset of CA curricula.

There is no doubt that suitable types of calculators can facilitate greatly the evaluation of polynomial functions such as the following cubic in which coefficients are understood to be rational: $f(n) = an^3 + bn^2 + cn + d$. At first sight it would seem that a calculator's $\boxed{y^x}$ key would be useful in connection with the n^3 factor of the first term. But use of this key triggers an internal processing that involves logs, and "$x \log y$" is undefined for $y < 0$. Yet it is highly likely that $f(n)$ is to be evaluated for values of $n < 0$ as well as for values of $n \geqslant 0$. What to do?

Some algebraic transformation of the function must be made so that use of the $\boxed{y^x}$ is circumvented. One possibility is $f(n) = an(n^2) + bn^2 + cn + d$, since n^2 can be evaluated in one or more ways without using a $\boxed{y^x}$ key. The preferred transformation vis-à-vis a calculator, however, is one involving a nested parentheses format: $f(n) = ((an + b)n + c)n + d$. This format generalizes readily to functions of higher degree--e.g., $f(n) = an^6 + bn^5 + cn^4 + dn^3 + en^2 + gn + h = (((((an + b)n + c)n + d)n + e)n + g)n + h$---and is especially effective as the basis for iterative calculator algorithms. For instance: See page 16 of this manuscript for one such illustration in which a repeated use of the $\boxed{y^x}$ key is circumvented effectively. (Although the HP-55 is programmable, a manually keystroked algorithm has been illustrated.)

This is but one of a nontrivial number of instances in which it is decidedly helpful, or even necessary, to modulate curricular content to conform to calculator characteristics (features, limitations, etc.). In these cases I refer to CA curricula as being more particularly CM curricula.

HP-55 User Instructions

Title $f(n) = an^6 + bn^5 + cn^4 + dn^3 + en^2 + gn + h$ ("Evaluation of" precedes it)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	Input/store coefficients	a	STO	6			
		b	STO	5			
		c	STO	4			
		d	STO	3			
		e	STO	2			
		g	STO	1			
		h	STO	0			
2	Input/store n_i	n_i	ENTER↑	ENTER↑	ENTER↑		
3	Calculate $f(n_i)$		RCL	6	×		
			RCL	5	+	×	
			RCL	4	+	×	
			RCL	3	+	×	
			RCL	2	+	×	
			RCL	1	+	×	
			RCL	0	+		$f(n_i)$
	For next n_i, repeat steps 2 & 3.						
	For new 6th-degree function, return to step 1.						

Calculator-based (CB) curricula. There is a sense in which any instruction pertaining to the calculator *itself* is calculator-based (CB). But I shall illustrate another sense in which CB curricula may be encountered.

Imagine students who have *no knowledge of logarithms* but who know that numbers such as 1, 10, 100, 1000, etc. can be expressed as integral powers of 10: 10^0, 10^1, 10^2, 10^3, etc., respectively. Imagine, too, that these students can use calculators having a $\boxed{10^x}$ key and that they are confronted with the following question (for the first time): Can a number such as 580 be expressed in exponential form to the base 10?

It is not far-fetched to expect that these students might reason that $100 < 580 < 1000$, or $10^2 < 580 < 10^3$; and since 580 is about half-way between 10^2 and 10^3, 580 should be about $10^{2.5}$. Use of key $\boxed{10^x}$ will show that $10^{2.5}$ is approximately 316.2278 (to four decimal places), which clearly is not "about" 580. But using key $\boxed{10^x}$ in connection with a guess-and-test procedure, the following sequence of increasingly more precise approximations may be generated rather quickly:

$$10^{2.7} < 580 < 10^{2.8}$$

$$10^{2.76} < 580 < 10^{2.77}$$

$$10^{2.763} < 580 < 10^{2.764}$$

$$10^{2.7634} < 580 < 10^{2.7635}$$

etc.

Work such as this should lay an excellent conceptual foundation for students when ultimately they keystroke "580 $\boxed{\log}$" and see displayed *2.76343* (to five decimal places).

The work just described clearly is calculator-*based* (CB). Instruction is more than assisted or modulated by the calculator: it is *dependent upon* the calculator; thus, CB.

The distinctions between CA, CM and CB curricula may be somewhat hazy at times, but that is not crucial. The distinctions are intended to emphasize that calculators may play somewhat different roles in relation to mathematics curricula, and that research should not be unmindful of such differential roles.

Prior and Ongoing Research

The very newness of calculators provides little of a research base upon which to build. For instance: Annotated research listings for the five calendar years 1970 through 1974 (Suydam & Weaver, 1971, 1972, 1973, 1974, 1975b) include a total of 444 journal reports and 1,443 doctoral dissertations that were judged to have relevance for mathematics education (although these two categories of documents are not disjoint). Only SIX investigations from the five years' listings pertained to calculators--see "References: Supplement A" --and most of these were concerned with desk-model rather than "pocket" or "hand-held" calculators.[3] When the annotated listing for calendar year 1975 is compiled, more calculator investigations are bound to appear; but there still will be no plethora of such investigations reported.

The extent of ongoing research is very difficult to assess; this also is true of the nature of that research. We are given hints from the brief progress reports released by some projects (e.g., Kessner, 1975; Barrett & Keefe, n.d., for the project announced by Hawthorne, 1973); but by and large we have precious little information--and none of it definitive--regarding the extent and nature of ongoing research.

Thus, my approach to a consideration of needed research on calculators in relation to school mathematics curricula has been literally from point zero. Over time we may see that findings from some of the ongoing research give at least partial answers to researchable questions that I shall raise. But at the present I have raised such questions without being able to build upon a body of findings from previous investigations, as we so often are able to do in many areas of mathematics education research.

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[3] The recently released monograph (Callahan & Glennon, 1975) that is the fourth edition of "A Guide to Current Research" pertaining to elementary school mathematics includes reference to no investigations involving calculators.

Some Needed Research

The National Advisory Committee on Mathematical Education (1975) has suggested that "Despite the obvious promise of calculators for enriching mathematics instruction, important questions of their optimal use must be investigated by thorough research:

> When and how should calculator use be introduced so that it does not block needed student understanding and skill in arithmetic operations and algorithms?
>
> Will ready access to calculators facilitate or discourage student memory of basic facts?
>
> For which mathematical procedures is practice with step-by-step paper and pencil calculation essential to thorough understanding and retention?
>
> What types of calculator design -- machine logic and display -- are optimal for various school uses?
>
> What special types of curricular materials are needed to exploit the classroom impact of calculators?
>
> How does calculator availability affect instructional emphasis, curriculum organization, and student learning styles in higher level secondary mathematics subjects like algebra, geometry, trigonometry, and calculus? [pp. 42-43]"

Other researchable questions are implicit in the NACOME report's discussion of calculators.

There will be some understandable overlap between the NACOME report questions (explicit and implicit) and those I now shall pose, although the two sets of questions were developed quite independently of each other.

1. In relation to students' work with mathematical problem situations and applications for which numerical calculations are necessary, what will be the effects of delaying (or eliminating?) the introduction, development and reinforcement of pencil-and-paper algorithms in favor of the introduction, development and reinforcement of calculator algorithms?

The improvement of students' ability to cope effectively with mathematical problem situations and applications is the raison d'être of school mathematics curricula. Computational facility *in and of itself* has no validity whatsoever as a curricular objective. (This is equally true of facility in algebraic and other symbolic manipulations.) Pencil-and-paper algorithms are means to ends, not ends in themselves. Are calculator algorithms "better" means to the same ends?

Figure 5a (manuscript page 21) suggests an all too commonplace practice which I shall illustrate within the context of whole-number multiplication. Assume that students have worked with factors less than 10 and now are confronted with multiplications in which one of the two factors is greater than nine. We draw upon A and B of Figure 5a to develop C--one or more pencil-and-paper algorithms--and then use C in connection with D. This is an unfortunate progression in that the focus of attention is upon C--use of an *algorithm*--rather than upon A--use of an *operation* and its properties (as needed). When we move on to instances in which each factor is greater than nine, we again draw upon A and B to develop C; then, in connection with D, the focus of attention is upon using *C*-- an *algorithm*. An unwanted consequence of all this is that students all too often very likely look upon these two problems

What is the cost of eight 13-cent stamps?

and

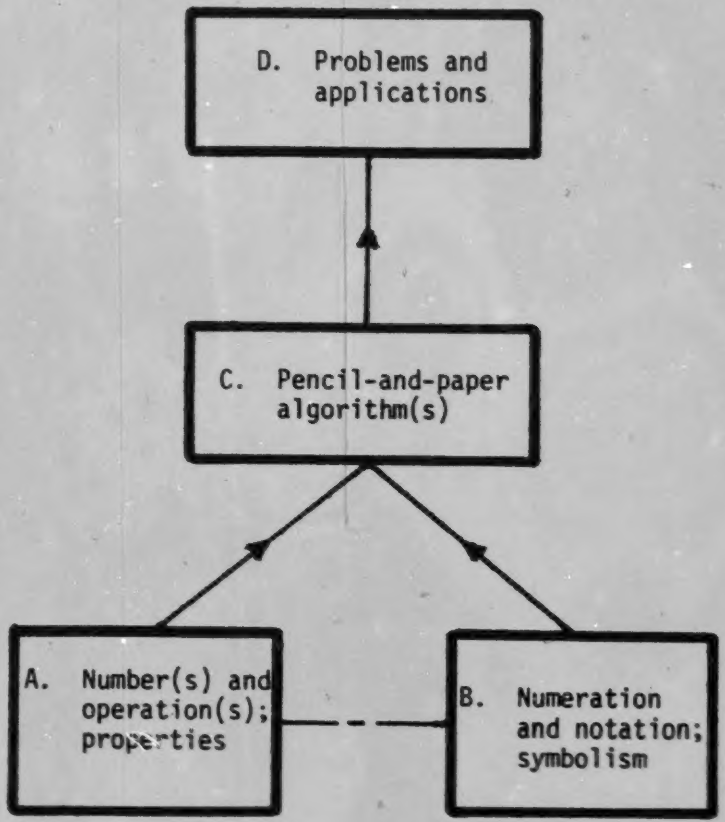
D. Problems and applications
C. Pencil-and-paper algorithm(s)
A. Number(s) and operation(s); properties
B. Numeration and notation; symbolism

Figure 5a. A commonplace practice

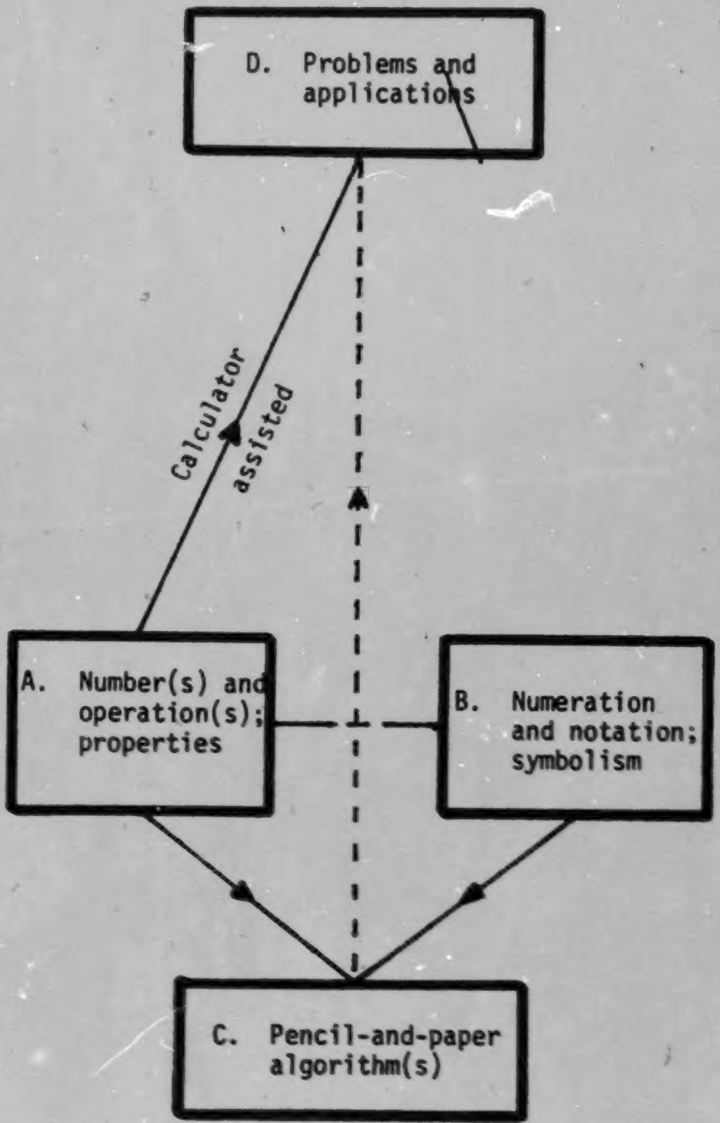
D. Problems and applications
Calculator assisted
A. Number(s) and operation(s); properties
B. Numeration and notation; symbolism
C. Pencil-and-paper algorithm(s)

Figure 5b. An alternative consideration

What is the cost of twenty-five 13-cent stamps?

as different--rather than alike--because somewhat different algorithms commonly would be used in effecting the multiplications.

If a particular calculator were used to solve the two problems, however, precisely the same calculator algorithm would be used for each problem; and that algorithm is so simple and direct that there is virtually nothing to divert students' attention from A to D directly. It is much more likely that students would view the two problems as being alike rather than different.

Figure 5b (manuscript page 21) suggests this desirable more direct relationship between A and D, with an option to introduce, develop and reinforce C to some degree at some point(s) in time with some students in connection with some curricular content, ultimately using C in connection with D.

Taken together, Figures 5a and 5b suggest a wide range of potential research on that which I consider to be a very crucial question pertaining to school mathematics curricula, a question with affective as well as cognitive implications, a question that must be related to the various components of Figure 1 (page 1 of this manuscript):

> In relation to students' work with mathematical problem situations and applications for which numerical calculations are necessary, what will be the effects of delaying, diminishing, or eliminating the introduction, development and reinforcement of pencil-and-paper algorithms in favor of the introduction, development and reinforcement of calculator algorithms?

But there is a hidden assumption in that question that raises another question --one that must be answered to some degree before making an extensive attack upon question #1.

---

2. How---and how readily and how well---can students learn to use calculators effectively and efficiently in connection with particular curricula?

The preceding question tacitly assumes that students will have little or no difficulty in learning to use calculators. This may or may not be true depending upon what is meant by using calculators. I believe this is a nontrivial point that needs more than passing consideration.

First, recall the different languages or logic schema that are used to enter and process data. The principal ones may be summarized as follows:

1 - Arithmetic or "commercial" logic (ArL)
2 - Algebraic logic (AgL)
    2.1 - With no operational hierarchy (AgL-No)
    2.2 - With a multiplication(or division)-before-addition(or subtraction) hierarchy (AgL-M/A)
3 - Reverse Polish notation (RPN) with an n-level operational stack

Certain of these language differences are reflected in even the simplest of calculator calculations. Refer, for instance, to equation types 1-4 of Table 1 (page 24 of this manuscript).

Given an AgL calculator (either AgL-No or AgL-M/A), the following routine or keystroke sequence is a generalized calculator algorithm applicable to equation types 1-4:

$$a \; \boxed{\nabla} \; b \; \boxed{=} .$$

Given an RPN calculator, a generalized algorithm for the same equation types is:

$$a \; \boxed{\text{ENTER}\uparrow} \; b \; \boxed{\nabla} .$$

There is no generalized algorithm for equation types 1-4, however, in the case of an ArL calculator.

TABLE 1

Some Simple Equation Types

I. $a \nabla b = m$	
1. $a + b = m$	
2. $a - b = m$	
3. $a \times b = m$	
4. $a \div b = m$	

II. $(a \nabla b) \# c = n$	III. $a \nabla (b \# c) = n$
5. $(a + b) + c = n$	21. $a + (b + c) = n$
6. $(a + b) - c = n$	22. $a + (b - c) = n$
7. $(a + b) \times c = n$	23. $a + (b \times c) = n$
8. $(a + b) \div c = n$	24. $a + (b \div c) = n$
9. $(a - b) + c = n$	25. $a - (b + c) = n$
10. $(a - b) - c = n$	26. $a - (b - c) = n$
11. $(a - b) \times c = n$	27. $a - (b \times c) = n$
12. $(a - b) \div c = n$	28. $a - (b \div c) = n$
13. $(a \times b) + c = n$	29. $a \times (b + c) = n$
14. $(a \times b) - c = n$	30. $a \times (b - c) = n$
15. $(a \times b) \times c = n$	31. $a \times (b \times c) = n$
16. $(a \times b) \div c = n$	32. $a \times (b \div c) = n$
17. $(a \div b) + c = n$	33. $a \div (b + c) = n$
18. $(a \div b) - c = n$	34. $a \div (b - c) = n$
19. $(a \div b) \times c = n$	35. $a \div (b \times c) = n$
20. $(a \div b) \div c = n$	36. $a \div (b \div c) = n$

Moving to equation types 6-36 (Table 1), more than calculator language or logic must be taken into consideration. I shall illustrate this with

$$625 - (19 \times 24) = n$$

which is a specific exemplar of equation type 27 and might have been derived from a "word problem" such as this:

> If I have 625 lollipops and sell 19 packs of 24 lollipops per pack, how many lollipops will I have left?

Without a calculator, a student very likely would execute a sequence of two pencil-and-paper calculations much as shown at the right in order to solve that equation or problem.

$$\begin{array}{r} 24 \\ \times\ \underline{19} \\ \underline{216} \\ \underline{240} \\ 456 \end{array} \qquad \begin{array}{r} 625 \\ \underline{-456} \\ 169 \end{array}$$

But how would a student solve the problem with a calculator? One obvious thing would be to follow the same two-step sequence, as it were, but to use a calculator to compute the product of 19 and 24, and then to subtract that product from 625.

Such a piecemeal approach would "work" on any calculator, but--depending upon the calculator--that approach may not be an efficient or preferred one. The piecemeal approach would involve re-entering the 456 via the keyboard, which, particularly in the case of large(r) numbers, increases the likelihood of visual and keystroking error. On certain calculators it is possible to avoid such re-entry of data by using particular calculator features such as an independent memory or storage register, or an operational stack, etc.

The intent of using a calculator involves more than simply replacing pencil-and-paper calculation with easy keystroking. Rather, the intent is to use a calculator algorithm that capitalizes upon any data processing features that a given calculator may have. Pages 26-27 of this manuscript illustrate such calculator algorithms for a variety of representative calculators.

An AgL-No calculator such as the Texas Instruments TI-2500-II which has no independent memory or storage register:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1		19	×				
2		24	=				456
3		625	-				
4		456	=				169

An AgL-No calculator such as the Texas Instruments TI-2550-II which has an independent "accumulating" memory:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1		625	CM	M+			
2		19	×				
3		24	=				456
4			M-	MR			169

An AgL-No calculator such as the Rockwell 18 R which has an independent "non-accumulating" or "overwrite" memory:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1		19	×				
2		24	=	STO			456
3		625	-	RCL	=		169

An AgL-No calculator such as the Litronix 2260 which has a pair of open- and close-parenthesis keys:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1		625	-	(			
2		19	×				
3		24	)				456
4			=				169

An AgL-M/A calculator such as the Texas Instruments SR-50 (illustrated by two algorithms which differ in whether the product of 19 and 24 is displayed):

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1		625	-				
2		19	×				
3		24	=				169

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1		625	STO				
2		19	×				
3		24	=				456
4			+/-	Σ	RCL		169

An RPN calculator such as the Hewlett-Packard HP-21:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1		625	ENTER↑				
2		19	ENTER↑				
3		24	×				456
4			-				169

No illustrative algorithm was shown for an ArL calculator on pages 26-27. This type of calculator seems to be on the decline, and rarely--if ever--has it been used as the base for constructing "scientific" or "slide-rule" calculators. Furthermore, in response to an article by Stultz (1975), both Gronbach (1975) and Weaver (1975) have seriously questioned the suitability of ArL calculators for instructional use, especially among young children.

A calculator-unique algorithm. The process of dividing is a familiar one that may be illustrated in the form shown at the right and defined more precisely as follows: Given (a,b), an ordered pair of whole numbers such that $b \neq 0$, the process of dividing specifies the whole numbers $\underline{q}$ and $\underline{r} < b$ so that $a = (b \times q) + r$.

$$\begin{array}{r} q \\ b\overline{)\ a} \\ -\underline{b\times q} \\ r \end{array}$$

Much research and discussion have been devoted to the pros and cons of sundry paper-and-pencil algorithms for this process, which is simplified markedly by use of a calculator.

Pages 29-31 illustrate various calculator algorithms for the process of dividing. Each algorithm is generated around these basic steps:

1. Calculate $a \div b = Q$.
2. Truncate Q to whole-number q: $[Q] = q$, which is in fact the "greatest integer" function applied to Q.
3. Calculate $b \times q$ (or $q \times b$).
4. Calculate $a - (b \times q) = r$

The truncation idea is rarely encountered by pupils in their pre-algebra work. It is a necessary idea, however, in *these* calculator algorithms (pages 29-31). Except for the last of the illustrative algorithms (HP-65 calculator), the truncation of Q to q is performed "mentally" by the pupil--not by the calculator. (More later about the HP-65.)

The illustrative algorithms differ principally in the extent to which, or way in which, calculator "memories" are used to advantage. They also differ in ways that are associated with differences between AgL-No, AgL-M/A, and RPN calculators.

Although the process of dividing is most commonly associated with "quotient and remainder" problem situations at the elementary-school level, the process has a more advanced application, for instance, in using the euclidean algorithm to generate the highest common factor of two counting numbers. The algorithm therefore is of interest in connection with more sophisticated as well as less sophisticated calculators.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
	TI-2500-II (Tex. Inst.)						
1		a	÷				
2		b	=				Q
3	Truncate: $[Q] = q$						
4		q	×				
5		b	=				bq
6		a	−				
7		bq	=				r
	TI-2550-II (Tex. Inst.)						
1		a	CM	M+	÷		
2		b	=				Q
3	Truncate: $[Q] = q$						
4		q	×				
5		b	=				bq
6			M−	MR			r
	18 R (Rockwell)						
1		a	÷				
2		b	STO	=			Q
3	Truncate: $[Q] = q$						
4		q	×	RCL	=	STO	bq
5		a	−	RCL	=		r

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
	SR-50 (Texas Instr.)						
1		a	STO	÷			
2		b	=				Q
3	Truncate: [Q] = q						
4		q	×				
5		b	=				bq
6			+/-	Σ	RCL		r
	SR-51A (Texas Instr.)						
1		a	STO	1	÷		
2		b	STO	2	=		Q
3	Truncate: [Q] = q						
4		q	×	RCL	2	=	bq
5			+/-	SUM	1		
6			RCL	1			r
	SR-52 (Texas Instr.)						
1		a	-	(	(		
2			RCL	÷			
3		b	STO	0	1	)	Q
4	Truncate: [Q] = q						
5		q	×	RCL	0	1	
6			)				bq
7			=				r

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
	HP-21 (Hewlett-Packard)						
1		a	ENTER↑	ENTER↑			
2		b	STO	÷			Q
3	Truncate: [Q] = q						
4			R↓				
5		q	RCL	×			bq
6			-				r
	HP-45 (Hewlett-Packard)						
1		a	ENTER↑	ENTER↑			
2		b	÷				Q
3	Truncate: [Q] = q						
4			R↓				
5		q	■	LAST x	×		bq
6			-				r
	HP-65						
1		a	ENTER↑	ENTER↑			
2		b	÷				Q
3			g	LST x	E		
4			f	INT			q
5			×				bq
6			-				r

It should be noted that in the HP-65 algorithm (bottom of page 31) the truncation of Q to q did not have to be effected "mentally" by the user but was executed using a hard-wired truncate-function key.

The SR-52 does not have such a hard-wired function key. But in connection with the SR-52 algorithm (bottom of page 30) it would be possible to eliminate "mental" truncation on the part of the user by inserting appropriately the following subroutine which has been adapted from an illustration in the SR-52 Owner's Manual:

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1	*	n	(	RCL	-		
2		.5	)	2nd	fix	0	
3			EE	INV	EE		
4			INV	2nd	fix		[n]
*	n > 0						

"Mental" truncation on the part of the user also could be eliminated in the case of any AgL calculator that does not have scientific notation by executing the following subroutine suggested by Smith (1975):

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
1		n	÷				
2	*	10000000···	×				
3	*	10000000···	=				[n]
*	Use a sufficient number of 0's to fill the display register.						

Younger users most likely would view this simply as a trick and would not comprehend its rationale.

Note that for all algorithms (pages 29-31) if Q is integral, q = Q and r = 0; thus, the rest of the algorithm is in fact superfluous.

An alternative algorithm. In connection with the process of dividing it is permissible to calculate r in the following way: $r = (Q - q) \times b$. The algorithms illustrated on pages 34-36 utilize this alternative calculation of r, which--depending upon the calculator involved--may show a nonintegral display that must be rounded off "mentally" by the user to get integral r.

(It may be of interest to note in passing that not only does the HP-25 calculator have a hard-wired function key to truncate Q to q, it also has a hard-wired function key to calculate $Q - q$ directly.)

Going a step further. Thus far, all illustrative algorithms are to be manually keystroked step-by-step by the user. Programming a calculator represents a still higher level of algorithm development. The appended "Illustrative Programs: II" utilize the programmable HP-65 and SR-52 calculators for the process of dividing. The two calculator programs per se differ in ways other than those observed earlier for the manually keystroked HP-65 and SR-52 algorithms, although the User Instructions for executing the two programs are almost identical.

"Illustrative Programs: III" and "Illustrative Programs: IV" use a different mathematical context to highlight further some of the programming differences between the HP-65 and SR-52 calculators.[4] "Illustrative Programs: IV" in particular shows an effective use of the SR-52's unique indirect addressing feature in connection with data storage and retrieval.

The question identified at the outset of this section,

"How---and how readily and how well---can students learn to use calculators effectively and efficiently in connection with particular curricula?",

is both broad and deep, and cannot be answered at all simply. The various illustrations used thus far barely scratch the surface of calculator features and differences that may have a bearing on this question, and that must be related to level of curricular content and to levels of students' cognitive and mathematical development. Our search for answers to this question must involve investigations that examine effects such as proactive and retroactive

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[4] This context, the evaluation of polynomial functions, was used earlier on manuscript pages 15-16 in connection with an illustrative HP-55 manually keystroked algorithm.

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
	TI-2500-II (Tex. Inst.)						
1		a	÷				
2		b	-				Q
3	Truncate: [Q] = q						
4		q	×				
5		b	=				r
	TI-2550-II (Tex. Inst.)						
1		a	÷				
2		b	CM	M+	-		Q
3	Truncate: [Q] = q						
4		q	×	MR	=		r
	18 R (Rockwell)						
1		a	÷				
2		b	STO	-			Q
3	Truncate: [Q] = q						
4		q	×	RCL	=		r

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
	SR-50 (Texas Instr.)						
1		a	÷				
2		b	STO	-			Q
3	Truncate: [Q] = q						
4		q	=	×	RCL	=	r
	SR-51A (Texas Instr.)						
1		a	÷				
2		b	STO	1	-		Q
3	Truncate: [Q] = q						
4		q	=	×	RCL	1	
5			=				r
	SR-52 (Texas Instr.)						
1			(				
2		a	÷				
3		b	STO	0	1	-	Q
4	Truncate: [Q] = q						
5		q	)	×	RCL		
6			0	1	=		r

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
	HP-21 (Hewlett-Packard)						
1		a	ENTER↑				
2		b	STO	÷			Q
3	Truncate: [Q] = q						
4		q	-	RCL	×		r
	HP-45 (Hewlett-Packard)						
1		a	ENTER↑				
2		b	÷				Q
3	Truncate: [Q] = q						
4			■	LAST x	x⇄y		
5		q	-	×			r
	HP-65 (Hewlett-Packard)						
1		a	ENTER↑				
2		b	÷				Q
3			g	LST x	E	ENTER↑	
4			f	INT	-	×	r

facilitation and interference, transfer, short- and long-term retention, etc. It also would be well to investigate learning within a "messing around" context as well as within the context of systematic instruction.

Elsewhere I have expressed the belief that students' intelligent and effective use of calculators will necessitate more rather than less mathematical comprehension (Weaver, 1975). Let us assume, for instance, that a student is aware of the following generalized algorithm for a calculator such as the TI-2550 when solving class III equation types (21-36) of Table 1 (page 24):

INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
b	#				
c	=	CM	M+		
a	∇	MR	=		n

For a particular exemplar of type 25 such as 81 - (15 + 29) this algorithm becomes:

INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
15	+				
29	=	CM	M+		
81	-	MR	=		37

But in the case of type 25 equations and exemplars, the following is a bit more efficient than the generalized algorithm used above:

INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
81	CM	M+			
15	+				
29	=	M-	MR		37

Furthermore, if a student is aware of the fact that a - (b + c) = (a - b) - c, that student might use a much more efficient algorithm for type 25 equations and exemplars:

INPUT DATA/UNITS	KEYS				OUTPUT DATA/UNITS
81	-				
15	-				
29	=				37

The preceding illustrations suggest that levels of knowledge about a calculator and levels of mathematical comprehension may influence how a student uses that calculator. We need to know much more than we do at present about the kind and degree of student knowledge and comprehension that facilitate more effective calculator use, that prompt students to modify calculator algorithms (or programs) when it would be advantageous to do so, and that enable students to devise or generate calculator algorithms (or programs) to meet particular needs.

Familiarity with certain mathematical ideas likely has a facilitating influence upon students' calculator use. But let us not overlook the likelihood of a chicken-and-egg question:

3. How can certain mathematical ideas (concepts, relationships, properties, and the like) be developed effectively and advantageously with the aid of a suitable calculator?

For several decades many members of the mathematics education community have adhered to the dictum, "Understanding precedes drill," and the related belief that drill in and of itself does not develop understanding. One of the classics from the research literature of the past focused on this dictum.[5]

I fear the emergence of a new dictum, "Understanding precedes calculator use," and the related contention that calculator use in and of itself does not develop mathematical understanding. I certainly agree with this latter contention. Although I sense the intent of the emerging new dictum, I cannot accept it as unqualifiedly valid.

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[5] W. A. Brownell & C. B. Chazal, The effects of premature drill in third-grade arithmetic. Journal of Educational Research, 1935, 29, 17-28.

I hold very strongly to the belief that calculators can be used to distinct advantage in the development of some mathematical ideas--that calculator use can precede and lead to understanding. One illustration of this already has been suggested on page 17 of this manuscript (pre-log experiences).

Consider another illustrative possibility: "Illustrative Programs: I", appended. A calculator program such as this might be used to advantage in developing the closure property as it pertains to the set of whole numbers and the operations of addition, subtraction, multiplication, and division.

Are there not a variety of properties that might be developed inductively through calculator use? Which ones? How?

4. What are the effects of implementing certain other calculator-inspired curricular changes?

Some curricular changes have been explicit or implicit in the questions posed thus far. But there are other changes that may merit investigation.

Certain of these have been suggested in a variety of documents: an earlier consideration of negative integers; an earlier consideration of decimal fractions (especially in relation to metrication), with less consideration of common fractions , and when coupled with the integers, an earlier consideration of the full set of rationals (decimal form emphasized); etc.

Such changes merit at least exploratory implementation and an effects assessment. But there are several others kinds of curricular changes that I believe merit consideration even though they are identified much less frequently --if at all.

_a_. I believe this to be a valid assertion: At the elementary-school level in particular, the sequencing and pacing of arithmetic content has been determined principally by the acquisition of computational skill--especially skill with pencil-and-paper algorithms. From a mathematical point of view it is absurd that _computational facility_ should play so large a role in the progressive introduction, development, and extension of content.

In effect I am touching upon another implication of Figures 5a and 5b (manuscript page 21). If we are guided by Figure 5b as a model, we can rely much more upon _conceptual_ rather than computational considerations in sequencing and pacing curricular content. Calculators can make it possible for us to consider seriously the implementation of promising changes in curricular sequencing and pacing not only at the elementary-school level, but also at the middle- and secondary-school levels as well.

b. Certain curricular changes may be more pedagogical than mathematical in nature. Earlier in this manuscript I illustrated within two different contexts a "guess-and-test" procedure. This certainly is a much more feasible tactic to use with a calculator than without one. Will the procedure improve students' ability to estimate, for instance? Can the procedure be used to develop systematic approaches to estimation?

Our consideration of calculator-inspired curricular changes and their effects should include pedagogical changes as well as mathematical ones.

Before leaving this question I would like to hoist at least one caution flag that takes us back to a content consideration. As I already indicated, it has been suggested frequently that calculators will necessitate less attention to rational number notation in common-fraction form. (Metrication also may prompt such decreased attention.)

There is danger, I believe, in unwittingly going too far in playing down common fractions. Decimal-fraction notation for rationals is not applicable within an algebraic context. If common-fraction notation for rationals is excessively suppressed in connection with arithmetic work, will students be handicapped in subsequent algebraic work? Why should not students use calculators in connection with operations on rationals that are expressed in common fraction form? (I have several calculator programs for just that situation!)

5. What about teacher education?

Consideration certainly must be given to this, especially at the elementary- and middle-school levels, within both preservice and inservice contexts.

What background must teachers have that will enable them to use calculators effectively in their instructional work? How can this background be attained expeditiously within preservice and inservice contexts?

6. What about calculators in relation to computers?

I know from personal experiences with secondary-school students and teachers that those who have worked with computers and computer programming are at a distinct advantage when working with calculators and calculator programming.

Is the converse true? Will persons who have worked with calculators and calculator programming have a "head start" for work with computers and computer programming? It is reasonable to believe so, but is that in fact the case? Is there any kind of proactive inhibition or interference (rather than facilitation) associated with calculator work prior to computer work?

Possibly these questions do not pertain so much to calculator vs. computer as they do to one level of computer vs. a higher one; for as Smith (1975) has indicated:

> From the standpoint that the programmable pocket calculator implements logical (Boolean) equations as well as Algebraic equations, can make logical decisions, and will iteratively execute a preprogrammed set of instructions, it can be correctly called a pocket computer. It is called a calculator only because it does not satisfy the U. S. Government's import/export trade definition of a computer. [p. 14]

Be that as it may, "References: Supplement B" of this position paper (pages 46-48) identify some research done on instructional uses of computers apart from (with one exception) CAI drill and practice. These references may suggest some research directions and approaches that should be considered in relation to calculators.

In Conclusion

I certainly have not exhausted the researchable questions that might be asked regarding the role of the hand-held electronic calculator in relation to school mathematics curricula. I have restricted the questions to those that I consider to be of more than mean importance, with less consideration for some that already are being raised frequently in a variety of sources.

Various kinds of research will need to be pursued. In some instances the investigations will be essentially feasibility studies; in other instances, experimental--although some of these would be premature at this point in time.

Some investigations will be within a CA context; some, within a CM context; and some within a CB-context--although the lines of demarcation may not always be clear. But that is of little consequence.

Not all investigations will emphasize the same component(s) of the research model identified as Figure 1; but all investigations should be sensitive to relevant components of the model.

Both cognitive and affective factors should be considered.

Some investigations should produce useful information in the relatively near future; others will require a longer period of time.

A Different Research Need

It has not been uncommon for past research pertaining to verbal problem solving, for instance, to include computational skill as a variable. A significant but low positive correlation generally is found to exist. It is of sufficient magnitude, however, to prompt investigators to "partial out" the influence of computational skill when examining the relationship of other factors (e.g., reading comprehension) to verbal problem solving ability.[6]

The use of calculators in connection with problem solving will literally eliminate as a source of variance the factor of computational skill involving pencil-and-paper algorithms. A new source of variance is introduced, to be sure; but of what significance?

Some past research on problem solving--and some other things as well--might profitably be redone, with the factor of computational skill now being of no research interest and not even entering into the investigation. Would new findings differ from previous ones? in what way(s), if at all?

It is one thing to control statistically for a variable; it is another thing to have the variable nonexistent! Let's put the calculator to this use.

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[6] For instance, see M. D. Martin, Reading, comprehension, abstract verbal reasoning, and computation as factors in arithmetic problem solving. (Doctoral dissertation, University of Iowa, 1963). Dissertation Abstracts, 1964, 24, 4547-4548.

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* This is mentioned here somewhat prematurely since it comes from the next JRME annual listing which has not been compiled to date.

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ILLUSTRATIVE PROGRAMS: I

THE CLOSURE PROPERTY
and OPERATIONS ON WHOLE NUMBERS

For every whole number m and every whole number n:

1. Is m + n a unique whole number? If not, when not?
2. Is m - n a unique whole number? If not, when not?
3. Is m × n a unique whole number? If not, when not?
4. Is m ÷ n a unique whole number? If not, when not?

Hewlett-Packard HP-65 and Texas Instruments SR-52 programs and user instructions to assist students to answer these questions follow. One aspect of these programs should be noted in particular.

The HP-65 calculator has a preprogrammed $\boxed{\text{INT}}$ function key that truncates an X-register number to an integer. This is used (program line 23) in connection with testing whether m ÷ n is a whole number.

The SR-52, however, has no such preprogrammed function. I have written the program to include alternative approaches to this:

a. In the user defined D-function to calculate and test m ÷ n, I found it advantageous to use the preprogrammed $\boxed{\text{D.MS}}$ (degrees-minutes-seconds) function key. This may appear irrelevant, even strange; but that key permitted me to devise a test based upon the fact that $x = x\boxed{\text{2nd}}\,\boxed{\text{D.MS}}$ iff x is an integer.

b. In the user defined D'-function to calculate and test m ÷ n, a longer subroutine was used to truncate $x > 0$ to an integer (and was bypassed in the event that $x = 0$). This truncation subroutine is derived from the SR-52 Owner's Manual.

The truncation of x to an integer is not an end in itself, but is a means to some other end which can be accomplished more efficiently (I believe) by use of the $\boxed{\text{D.MS}}$ key. It would be a quite different matter, however, if the truncation of x to an integer were an end in itself (which is not accomplished by use of the $\boxed{\text{D.MS}}$ key.)

J. F. Weaver

The program was NOT written (altho it could have been) to detect & reject non-whole number inputs.

HP-65 Program Form

Title m # n within the whole-number universe

Page 1 of 2

SWITCH TO W/PRGM PRESS [f] [PRGM] TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
LBL	23	
A	11	
+	61	
RTN	24	
LBL	23	
B	12	
-	51	
0	00	
g x>y	35 24	
÷	81	
g NOP	35 01	
g R↓	35 08	
RTN	24	
LBL	23	
C	13	
×	71	
RTN	24	
LBL	23	
D	14	
÷	81	
ENTER↑	41	
f	31	
INT	83	
g x≠y	35 21	
CL x	44	
÷	81	
RTN	24	

KEY ENTRY	CODE SHOWN	COMMENTS

REGISTERS

R$_1$

R$_2$

R$_3$

R$_4$

R$_5$

R$_6$

R$_7$

R$_8$

R$_9$

LABELS

A	m + n
B	m - n
C	m × n
D	m ÷ n
E	
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS

1

2

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TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 User Instructions

54

Title m # n within the whole-number universe

Page 2 of 2

Programmer jfw

Date 12/75

+	-	×	÷	

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	To add: m + n	m	ENTER↑		
		n §	A		m + n
	OR				
2	To subtract: m - n	m	ENTER↑		
		n §	B		m - n †
	OR				
3	To multiply: m × n	m	ENTER↑		
		n §	C		m × n
	OR				
4	To divide: m ÷ n	m	ENTER↑		
		n §	D		m ÷ n †
	† A flashing or blinking display signifies that no m # n exists, or that no unique m # n exists, among the whole numbers				
	§ Iff n = m it is not necessary to enter n via the keyboard and this entry may be omitted				

SR-52 Coding Form

TITLE m # n within the whole-number universe PAGE 1 OF 2
PROGRAMMER jfw DATE 12/75
An asterisk * signifies use of the prefix key [2nd]

LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS
000	46	*LBL		038	75	-					
001	11	A		039	90	*if zro					
002	95	=		040	10	*E'					
003	81	HLT		041	42	STO					
004	46	*LBL		042	00	0		080 192			
005	12	B		043	01	1					
006	95	=		044	53	(					
007	22	INV		045	43	RCL					
008	80	*if pos		046	75	-					
009	15	E		047	93	.		085 197			
010	81	HLT		048	05	5					
011	46	*LBL		049	54	)					
012	13	C		050	57	*fix					
013	95	=		051	00	0					
014	81	HLT		052	52	EE		090 202			
015	46	*LBL		053	22	INV					
016	14	D		054	52	EE					
017	75	-		055	22	INV					
018	42	STO		056	57	*fix					
019	00	0		057	95	=		095 207			
020	01	1		058	22	INV					
021	37	*D.MS		059	90	*if zro					
022	95	=		060	15	E					
023	22	INV		061	43	RCL					
024	90	*if zro		062	00	0		100 212			
025	15	E		063	01	1					
026	43	RCL		064	46	*LBL					
027	00	0		065	10	E'					
028	01	1		066	81	HLT					
029	81	HLT						105 217			
030	46	*LBL									
031	15	E									
032	55	÷		070 182							
033	00	0									
034	95	=						110 222			
035	81	HLT									
036	46	*LBL	Alternate								
037	19	*D'	for D	075 187							

LABELS	
A	m + n
B	m - n
C	m × n
D	m ÷ n
E	Used
A'	
B'	
C'	
D'	
E'	
REGISTERS	
00	
01	m ÷ n
02	
03	
04	
05	
06	
07	
08	
09	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
FLAGS	
0	
1	
2	
3	
4	

TEXAS INSTRUMENTS
INCORPORATED

SR-52 User Instructions

TITLE m # n within the whole-number universe PAGE 2 OF 2

◄A►				
+	-	×	÷	

◄B►				

STEP	PROCEDURE	ENTER	PRESS			DISPLAY
1	To add: m + n	m	+			
		n §	A			m + n
	OR					
2	To subtract: m - n	m	-			
		n §	B			m - n †
	OR					
3	To multiply: m × n	m	×			
		n §	C			m × n
	OR					
4	To divide: m ÷ n	m	÷			
		n §	D ¶			m ÷ n †
	† A flashing or blinking					
	display signifies that no					
	(unique) m # n exists					
	among the whole numbers					
	§ Iff n = m it is permissible					
	to use RCL or STO in-					
	stead of entering n via					
	the keyboard					
	¶ Alternative execution:		2nd	D'		

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ILLUSTRATIVE PROGRAMS: II

THE PROCESS OF DIVIDING

Given the ordered pair of whole numbers (a,b), b ≠ 0, determine whole numbers q and r such that a = bq + r, r < b.

HP-65 and SR-52 programs and user instructions follow. In each program q is calculated by first calculating a ÷ b = Q and then truncating rational Q to integral q. Then r is calculated as a - bq.

$$\begin{array}{r} q \\ b\overline{)\ a} \\ \underline{-bq} \\ r \end{array}$$

Each program is designed to permit:

(1) a and b to be entered in either order (a, then b; or b, then a); and

(2) bq to be displayed optionally before r, or after r, or not at all.

(Shorter programs could be written if one or both of these restrictions were not in effect.)

If an illegal b = 0 is used as input, the display will BLINK or FLASH when key $\boxed{C}$ is depressed to calculate q.

The programs have NOT been written to detect and reject non-whole number inputs (although such could have been done).

J F Weaver

HP-65 Program Form

Title The Process of Dividing

Page 1 of 2

SWITCH TO W/PRGM PRESS f PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
DSP	21	
.	83	
0	00	
R/S	84	
LBL	23	
A	11	
STO 1	33 01	
RTN	24	
LBL	23	
B	12	
STO 2	33 02	
RTN	24	
LBL	23	
C	13	
RCL 1	34 01	
RCL 2	34 02	
÷	81	
f	31	
INT	83	
STO 3	33 03	
RTN	24	
LBL	23	
D	14	
RCL 2	34 02	
RCL 3	34 03	
×	71	
RTN	24	
LBL	23	
E	15	
RCL 1	34 01	
D	14	
-	51	
RTN	24	

REGISTERS	
R_1	a
R_2	b
R_3	q
R_4	
R_5	
R_6	
R_7	
R_8	
R_9	

LABELS	
A	a in
B	b in
C	q
D	bq
E	r
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

FLAGS	
1	
2	

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TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 User Instructions

60

Title The Process of Dividing

Page 2 of 2

Programmer jfw

Date 12/75

a in	b in	q	bq	r

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
0	Initialize for 0 decimal places		RTN	R/S	
1	Input a and b in any order	a	A		a
		b	B		b
2	Calculate q		C		q *
3	OPTIONAL: Display bq		D		bq
4	Calculate r		E		r
5	OPTIONAL: Display bq		D		bq
	For a new problem, return to step 1				
	* If display BLINKS or FLASHES, b is invalid and no (unique) quotient exists. Return to step 2 with a new b or to step 1 for a new problem.				

TITLE The Process of Dividing PAGE 1 OF 2
PROGRAMMER jfw DATE 12/75

SR-52 Coding Form

An asterisk * signifies use of the prefix key 2nd .

LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LABELS
000 112	46	*LBL		038	00	0						A a in
001	11	A		039	03	3						B b in
002	42	STO		040 152	81	HLT						C q
003	00	0		041	46	*LBL						D bq
004	01	1		042	15	E		080 192				E r
005 117	81	HLT		043	43	RCL						A'
006	46	*LBL		044	00	0						B'
007	12	B		045 157	01	1						C'
008	42	STO		046	75	-						D'
009	00	0		047	46	*LBL		085 197				E'
010 122	02	2		048	14	D						REGISTERS
011	81	HLT		049	43	RCL						00
012	46	*LBL		050 162	00	0						01 a
013	13	C		051	02	2						02 b
014	43	RCL		052	65	×		090 202				03 q
015 127	00	0		053	43	RCL						04
016	01	1		054	00	0						05
017	55	÷		055 167	03	3						06
018	43	RCL		056	95	=						07
019	00	0		057	81	HLT		095 207				08
020 132	02	2										09
021	95	=										10
022	90	*if zro		060 172								11
023	18	*C'										12
024	75	-						100 212				13
025 137	93	.										14
026	05	5										15
027	95	=		065 177								16
028	57	*fix										17
029	00	0						105 217				18
030 142	52	EE										19
031	22	INV										FLAGS
032	52	EE		070 182								0
033	22	INV										1
034	57	*fix						110 222				2
035 147	46	*LBL										3
036	18	*C'										4
037	42	STO		075 187								

TEXAS INSTRUMENTS
INCORPORATED

SR-52 User Instructions

TITLE The Process of Dividing PAGE 2 OF 2

◀A◀				
a in	b in	q	bq	r

◀B◀				

STEP	PROCEDURE	ENTER	PRESS			DISPLAY
1	Input a & b (any order)	a	A			a
		b	B			b
2	Calculate q		C			q *
3	OPTIONAL: Display bq		D			bq
4	Calculate r		E			r
5	OPTIONAL: Display bq		D			bq
	For a new problem, return					
	to step 1					
	* If display BLINKS or					
	FLASHES, b is invalid;					
	no (unique) quotient					
	exists. Return to step					
	2 with a new b or to					
	step 1 for a new problem					

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Part No. [illegible]

ILLUSTRATIVE PROGRAMS: III

GENERATING DATA POINTS
FOR THE GRAPH OF THE FUNCTION
$y = ax^3 + bx^2 + cx + d$

HP-65 and SR-52 programs and user instructions follow. The programs are written so that the coefficients a, b, c, d may be entered in any order.

Calculations are based upon an equivalent form of the function:

$y = ((ax + b)x + c)x + d$.

J F Weaver

HP-65 Program Form

Title Data Points for Graph of y = ax^3 + bx^2 + cx + d

Page 1 of 2

SWITCH TO W/PRGM PRESS f PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS
LBL	23	
A	11	
STO 3	33 03	
RTN	24	
LBL	23	
B	12	
STO 2	33 02	
RTN	24	
LBL	23	
C	13	
STO 1	33 01	
RTN	24	
LBL	23	
D	14	
STO 5	33 05	
RTN	24	
LBL	23	
E	15	
ENTER↑	41	
ENTER↑	41	
ENTER↑	41	
RCL 3	34 03	
×	71	
RCL 2	34 02	
+	61	
×	71	
RCL 1	34 01	
+	61	
×	71	
RCL 5	34 05	
+	61	
RTN	24	

KEY ENTRY	CODE SHOWN	COMMENTS

REGISTERS

R$_1$ c

R$_2$ b

R$_3$ a

R$_4$

R$_5$ d

R$_6$

R$_7$

R$_8$

R$_9$

LABELS

A a in

B b in

C c in

D d in

E y$_1$

0

1

2

3

4

5

6

7

8

9

FLAGS

1

2

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 User Instructions

Title Data Points for the Graph of $y = ax^3 + bx^2 + cx + d$ Page 2 of 2

Programmer jfw Date 12/75

a in	b in	c in	d in	y_i

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Input a,b,c,d in any order	a	A		a
		b	B		b
		c	C		c
		d	D		d
2	Input x_i; calculate y_i	x_i	E		y_i
	Repeat step 2 for each (x_i, y_i) desired				
	For a new function, return to step 1				

TITLE Data Points for Graph of $y = ax^3 + bx^2 + cx + d$ PAGE 1 OF 2
PROGRAMMER jfw DATE 12/75

SR-52 Coding Form

An asterisk * signifies use of the prefix key 2nd.

LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS
000 112	46	*LBL		038	02	2					
001	11	A		039	54	)					
002	42	STO		040 152	65	×					
003	00	0		041	43	RCL					
004	03	3		042	00	0		080 192			
005 117	81	HLT		043	04	4					
006	46	*LBL		044	85	+					
007	12	B		045 157	43	RCL					
008	42	STO		046	00	0					
009	00	0		047	01	1		085 197			
010 122	02	2		048	54	)					
011	81	HLT		049	65	×					
012	46	*LBL		050 162	43	RCL					
013	13	C		051	00	0					
014	42	STO		052	04	4		090 202			
015 127	00	0		053	85	+					
016	01	1		054	43	RCL					
017	81	HLT		055 167	00	0					
018	46	*LBL		056	00	0					
019	14	D		057	95	=		095 207			
020 132	42	STO		058	81	HLT					
021	00	0									
022	00	0		060 172							
023	81	HLT									
024	46	*LBL						100 212			
025 137	15	E									
026	53	(									
027	53	(		065 177							
028	42	STO									
029	00	0						105 217			
030 142	04	4									
031	65	×									
032	43	RCL		070 182							
033	00	0									
034	03	3						110 222			
035 147	85	+									
036	43	RCL									
037	00	0		075 187							

LABELS	
A	a in
B	b in
C	c in
D	d in
E	y_1
A'	
B'	
C'	
D'	
E'	

REGISTERS	
00	d
01	c
02	b
03	a
04	x_1
05	
06	
07	
08	
09	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	

FLAGS	
0	
1	
2	
3	
4	

TEXAS INSTRUMENTS
INCORPORATED

SR-52
User Instructions

TITLE Data Points for the Graph of $y = ax^3 + bx^2 + cx + d$ PAGE 2 OF 2

A				
a in	b in	c in	d in	y_i

B				

STEP	PROCEDURE	ENTER	PRESS			DISPLAY
1	Input a,b,c,d in any order	a	A			a
		b	B			b
		c	C			c
		d	D			d
2	Input x_i; calculate y_i	x_i	E			y_i
	Repeat step 2 for each					
	(x_i, y_i) desired					
	For a new function, return					
	to step 1					

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ILLUSTRATIVE PROGRAMS: IV

EVALUATION OF POLYNOMIAL FUNCTIONS

Given a particular polynomial function of the form

$$f(x) = c_n x^n + c_{n-1} x^{n-1} + \cdot \cdot \cdot + c_2 x^2 + c_1 x + c_0$$

with rational coefficients (c_j's) and n a positive integer, that function may be evaluated for selected rational x_i's using the attached SR-52 program, valid for $0 < n < 18$, which is based upon an equivalent form of the function:

$$f(x) = (((c_n x + c_{n-1})x + \cdot \cdot \cdot + c_2)x + c_1)x + c_0.$$

The program was designed so that the complete store/recall of c_j's would be embedded within user-defined functions (without manually keystroking any register addresses), using to advantage the SR-52's indirect-instructions feature.

Also attached is a somewhat similar HP-65 program that, in view of fewer addressable registers and no indirect-instructions provision, is limited to the range $0 < n < 7$. This may be extended by manual keystroking, however, to $n > 6$.

J F Weaver

TITLE Evaluation of Polynomial Functions PAGE 1 OF 2

PROGRAMMER J F Weaver DATE 12/75

* signifies use of prefix key 2nd

SR-52 Coding Form

LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS	LOC	CODE	KEY	COMMENTS
000	46	*LBL		038	01	1					
001	11	A		039	08	8					
002	47	*CMs		040	14	D					
003	85	+		041	42	STO					
004	01	1		042	00	0		080 192			
005	75	-		043	00	0					
006	42	STO		044	41	GTO					
007	00	0		045	89	*3'					
008	00	0		046	46	*LBL					
009	01	1		047	88	*2'		085 197			
010	95	=		048	65	×					
011	46	*LBL		049	43	RCL					
012	87	*1'		050	01	1					
013	81	HLT		051	08	8					
014	46	*LBL		052	85	+		090 202			
015	13	C		053	46	LBL					
016	36	*IND		054	89	*3'					
017	42	STO		055	36	*IND					
018	00	0		056	43	RCL					
019	00	0		057	00	0		095 207			
020	65	×		058	00	0					
021	01	1		059	95	=					
022	44	SUM		060	58	*dsz					
023	01	1		061	88	*2'					
024	09	9		062	81	HLT		100 212			
025	95	=									
026	58	*dsz									
027	87	*1'		065 177							
028	81	HLT									
029	46	*LBL						105 217			
030	14	D									
031	43	RCL									
032	01	1		070 182							
033	09	9									
034	56	*rtn						110 222			
035	46	*LBL									
036	15	E									
037	42	STO		075 187							

LABELS	
A	Initial.
B	
C	c_j's In
D	# of c_j's
E	$f(x_i)$
A'	
B'	
C'	
D'	
E'	
REGISTERS	
00	dsz
01	c_0
02	c_1
03	c_2
04	c_3
05	etc. thru c_n ↓
06	
07	
08	
09	
10	
11	
12	
13	
14	
15	
16	
17	
18	x_i
19	# of c_j's
FLAGS	
0	
1	
2	
3	
4	

TEXAS INSTRUMENTS
INCORPORATED

SR-52 User Instructions

TITLE Evaluation of Polynomial Functions $(0 < n < 18)$ PAGE 2 OF 2

☒ ←A→				
Init.		In c_j's	# of c_j's	$f(x_i)$

☐ ☐ ←B→				

STEP	PROCEDURE	ENTER	PRESS			DISPLAY
1	Enter program					
2	Initialize (calculate mode)	n	A			n
3	Input/store coefficients	c_n	C	or	RUN	c_n
	in order, including any for	c_{n-1}	C	or	RUN	c_{n-1}
	which $c_j = 0$	....	C	or	RUN	....
		c_2	C	or	RUN	c_2
		c_1	C	or	RUN	c_1
		c_0	C	or	RUN	c_0
4	OPTIONAL check: Number of		D			$n+1$
	c_j's, step 3					
5	Input x_i; compute $f(x_i)$	x_i	E			$f(x_i)$
	Repeat 5 for each x_i desired					
6	For a new function return					
	to step 2					

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HP-65 Program Form

Title Evaluation of Polynomial Functions Page 1 of 2

SWITCH TO W/PRGM PRESS f PRGM TO CLEAR MEMORY

KEY ENTRY	CODE SHOWN	COMMENTS	KEY ENTRY	CODE SHOWN	COMMENTS
LBL	23		DSZ	83	
C	13		GTO	22	
STO 1	33 01		3	03	
B	12		RTN	24	
R/S	84		LBL	23	
STO 2	33 02		3	03	
B	12		×	71	
R/S	84		RCL 3	34 03	
STO 3	33 03		+	61	
B	12		g	35	
R/S	84		DSZ	83	
STO 4	33 04		GTO	22	
B	12		4	04	
R/S	84		RTN	24	
STO 5	33 05		LBL	23	
B	12		4	04	
R/S	84		×	71	
STO 6	33 06		RCL 4	34 04	
B	12		+	61	
R/S	84		g	35	
STO 7	33 07		DSZ	83	
LBL	23		GTO	22	
B	12		5	05	
1	01		RTN	24	
STO	33		LBL	23	
+	61		5	05	
9	09		×	71	
g R↓	35 08		RCL 5	34 05	
RTN	24		+	61	
LBL	23		g	35	
E	15		DSZ	83	
ENTER↑	41		GTO	22	
ENTER↑	41		6	06	
ENTER↑	41		RTN	24	
RCL	34		LBL	23	
9	09		6	06	
STO 8	33 08		×	71	
CL x	44		RCL 6	34 06	
RCL 1	34 01		+	61	
g	35		g	35	
DSZ	83		DSZ	83	
GTO	22		GTO	22	
2	02		7	07	
RTN	24		RTN	24	
LBL	23		LBL	23	
2	02		7	07	
×	71		×	71	
RCL 2	34 02		RCL 7	34 07	
+	61		+	61	
g	35		RTN	24	

REGISTERS	
R_1	c_n
R_2	c_{n-1}
R_3	c_{n-2}
R_4	c_{n-3}
R_5	c_{n-4}
R_6	c_{n-5}
R_7	c_{n-6}
R_8	DSZ
R_9	Used number of c_j's

LABELS	
A	
B	Used
C	c_j's In
D	
E	$f(x_i)$
0	
1	
2	Used
3	Used
4	Used
5	Used
6	Used
7	Used
8	
9	

FLAGS	
1	
2	

295

9320-0616

TO RECORD PROGRAM INSERT MAGNETIC CARD WITH SWITCH SET AT W/PRGM

HP-65 User Instructions

Title Evaluation of Polynomial Functions $(0 < n < 7)$ Page 2 of 2

Programmer J F Weaver Date 12/75

		In c_j		$f(x_i)$

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS		OUTPUT DATA/UNITS
1	Enter program				
2	Initialize in RUN mode		f	REG	
3	Input/store c_j's in order, including any for	c_n	C		c_n
	which $c_j = 0$	c_{n-1}	R/S		c_{n-1}
		....	R/S		....
		c_2	R/S		c_2
		c_1	R/S		c_1
		c_0	R/S		c_0
4	OPTIONAL check: Number of c_j's, step 3		RCL	9	$n + 1$
5	Input x_i; compute $f(x_i)$	x_i	E		$f(x_i)$
	Repeat step 5 for each x_i desired				
6	For a new function, return to step 2				
	For $n > 6$, use step 3 for c_n thru c_{n-6}. Then				
	after executing step 5 for a particular x_i				
	continue calculation of $f(x_i)$ for that x_i				
	using the following manual keystroking		×		
		c_{n-7}	+		
			×		
		c_{n-8}	+		
		....			
			×		
		c_0	+		$f(x_i)$

APPENDIX F

CALCULATORS AND SCHOOL ARITHMETIC: Some Perspectives

by

Zalman Usiskin and Max Bell

The University of Chicago

February, 1976

Preface

This paper, as its title suggests, deals with the possible roles of hand-held calculators in the calculation curriculum in grades K-8. It has been written at the request of and with the support of the Electronic Hand-Held Calculator Project directed by Marilyn Suydam and Richard Shumway of Ohio State University but it could not have been written without the additional experience we gained through two NSF-funded Projects under our direction at the University of Chicago: "Explorations into Ways of Improving the Elementary Mathematics Learning Experience," and "First-Year Algebra via Applications Development Project."

The main body of this paper is divided into three connected parts. There are also three appendices giving our views on important issues related to the main body of the paper.

We wish to acknowledge the invaluable assistance of Katherine Blackburn in collecting some of the data we used for this paper.

The University of Chicago
February, 1976

Max Bell and Zalman Usiskin

Calculators and School Arithmetic: Some Perspectives

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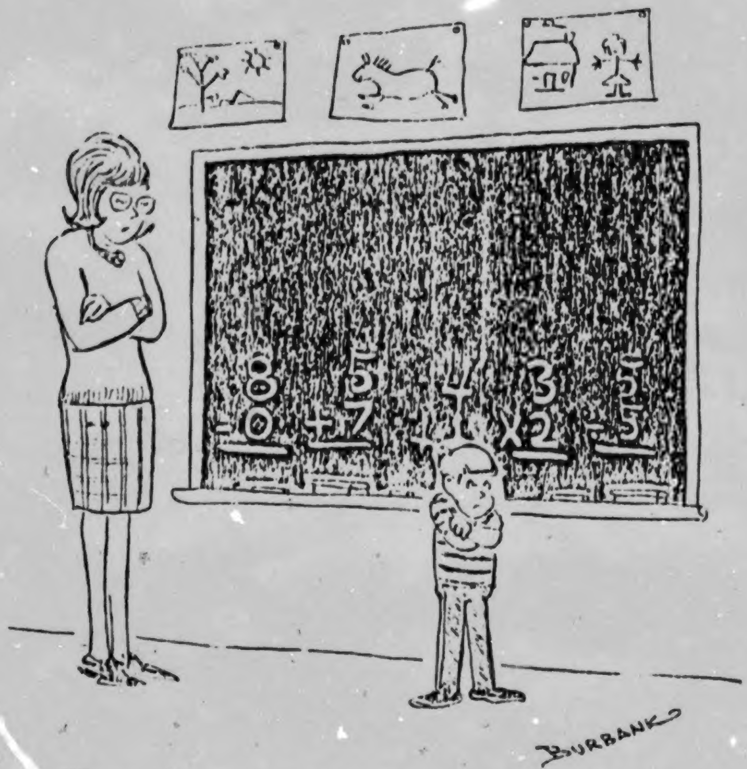

"Because batteries go dead in pocket calculators, that's why."

Saturday Review
January 24, 1976

CALCULATORS AND SCHOOL ARITHMETIC: Some Perspectives

Part I

A Review of the Present Situation With Respect to Calculators, Calculation, and the Elementary School Calculation Curriculum

A. The availability of calculators and some possible reactions to that availability

In an assessment perhaps eighteen years too soon, a writer in The Arithmetic Teacher had this to say:

> Skills in computation are no longer essential. Calculating machines are available to everyone at prices which are in keeping with their needs. The introduction of the calculating machine has increased rather than lessened the importance of learning arithmetic. The need for extreme skill and speed, however, in obtaining "answers" has disappeared. The demand is now for the understanding of arithmetic, its logical structure, and its practical social application. (Schott, 1957)

That assertion of availability of calculating machines for everyone was certainly inaccurate in 1957, but the situation in that respect has changed radically within just a few years. The first hand-held electronic calculators appeared only in 1971 at prices in the hundreds of dollars. Only three years later the New York Times (Nash, 1974) reported that one in ten Americans already owned calculators and projected a saturation of the world market by 1980 with 160 million units in use with 40 million units sold per year thereafter (based on a four year life). The article did not indicate how many of these sales would be in the U.S. but projections from industry sources are generally similar to those below, which are from a 1975 survey for industry by a reliable private research organization.

Table 1: Projections of Calculator Sales in the United States

	Hand-held 4 function (millions)	All other hand-held (millions)	Total (millions)	Cumulative Totals (millions)	Average per unit selling price
1973	7.70	.64	8.34	8.34	$85
1974	9.65	.87	10.52	18.86	$53
1975	11.60	1.17	12.77	31.63	$41
1976	13.60	1.45	14.75	46.38	$35
1980	15.80	2.40	18.20	112.50*	$25

*Our extrapolation, assuming sales of about 15, 16, and 17 million in 1977, 1978, 1979 respectively. Suydam and Shumway, 1975, give less conservative figures.

Allowing for loss, obsolescence, etc. it seems fair to project from these figures that perhaps 35 to 40 million calculators might be in use by the end of 1976 and a conservative estimate might be 65 to 75 million by the end of 1980.* (In 1975 there were about 70-million households in the United States.)

Assuming such projections prove to be accurate, we can conclude that whatever the response of schools, people outside of schools will have nearly universal access to cheap and easy calculation by 1980, which seems to be the very earliest that any curriculum changes based on calculator availability could have much effect.

---

*It is interesting to note that the preponderance of calculators sold are general consumer-oriented machines.

The likelihood of nearly universal access to cheap calculators has already generated considerable attention in the news media, with the question of effects on schools prominent in nearly all media coverage. So much has been said and printed by now on the supposed "issues" thus generated and so much of it is repetitious that it would serve little purpose to review that media coverage here beyond saying that two polar views are given more or less equal prominence in most of the coverage to date. They are well expressed in this excerpt from Nation's Schools and Colleges (December 1973):

> Critics argue that students, especially at elementary levels, risk becoming so dependent on calculators that they will forget - or fail to learn in the first place - basic computational skills. Moreover, the critics say, high calculator costs often place the devices out of reach of the economically disadvantaged. Proponents challenge both points. They contend mini-calculators can be a significant force in moving schools away from "answer-oriented" instruction, freeing both teachers and students for concentration on more important underlying concepts.

The issues summarized in brief by the quotation above and in more detail in the Suydam and Shumway paper (see bibliography) are genuine ones that can presumably be dealt with over the next few years by research, curriculum development and trial, varied approaches in different places, and compromise. (Can be doesn't necessarily say will be, of course.) But there is also plenty of scope in the issues presented by calculators for determined and passionate argument on "moral" grounds, and that sort of controversy can obscure the educational and pedagogical issues. For example, such "issues" as these have been raised in newspaper editorials (e.g., *The Chicago Tribune* and *The Wall Street Journal*) and in comment from a variety of earnest people (including many teachers): calculators reinforce dependence on machines (technology) and hence reduce humanity; hard things shouldn't be made too easy; giving calculators to students who haven't demonstrated computational skill rewards sloth and ignorance; calculators in schools amounts to pampering, frills, waste of taxpayers money, and other such code words for certain standard concerns about schooling.

Statements of such concerns often resemble the more substantive pedagogical arguments about the possible ill effects of failure to learn computation, but the quality and intent of them is quite different. Calculator use in schools could easily become a battleground around such "moral" assumptions, and this is especially so with respect to use of calculators in early schooling and by those older students unable to demonstrate good calculation skills. This could happen regardless of the utility of using calculators as aids to learning mathematics. Indeed there appears to be developing a possibly defensive concensus among educators that calculators must "of course" not be allowed in schools until calculation skill is proved--a concensus based on no inquiry whatever and very likely influenced mainly by a wish to minimize controversy. There is probably little we can do to head off such controversy around various moral assumptions in the world at large, but it is certainly not too early to attempt an assessment of the important pedagogical issues and possibilities and to set in motion a program of well designed investigation of them.

While one can't predict with certainty what the long range effects of calculators on school and society will be, it seems to us probable that these effects will be more revolutionary than not. Many years ago Hamming noted in an article entitled "Intellectual Implications of the Computer Revolution" (Hamming, 1963) that "It is a common observation that a change of an order of magnitude in a technology produces fundamentally new effects." Viewed from several perspectives, calculators represent at least an order of magnitude change in calculating power available to nearly everyone so perhaps we should anticipate fundamentally new effects, at least in the society at large.*

There is ample precedent for significant discontinuity between school and society and it may well be that even if there were such revolutionary changes in society, they would not be reflected, or reflected only after many years, in what goes on in schools. Mathematics education as a profession should try to assure that this particular gap between school and society does not develop by default; that is, merely for lack of energy and thoughtfulness in defining and investigating the possibilities. In any such attempt to define possibilities for calculators in schools, it would be foolish to ignore the early school mathematics experience of youngsters. The possible impact of calculators on the calculation component of that early experience (grades K-8) is our focus in this paper.

---

*A letter in Datamation some time ago characterized the first Hewlett Packard programmable hand-held calculator as essentially a hand-held ENIAC--one of the first of the practical electronic computers. Hence the imminent democratization of calculating power may fairly soon become democratization of true computer power for anyone able to learn to use such power, a possibility that educators have not even begun to consider, and that we will not consider here.

B. An historical perspective on calculation skills

In 1956 (before the so-called "New Math") John Gardner characterized the state of mathematics education as "A National Weakness" in the following words:

> Concern over the mathematical incompetence of the average--and even above-average--American has become almost a national preoccupation. Science and industry cry in vain for more and better mathematicians. Ordinary businesses ask only that their employees be able to do simple arithmetic. Neither the extravagant nor the modest demands of society for mathematicians--or arithmeticians even--are being met. And the public concern grows... (Gardner, 1956)

At about the same time college and university teachers of mathematics were in despair at the poor mathematics preparation of entering students. "Why Johnny can't add" is hardly a new concern.

Whatever else may be said of the reforms accomplished since 1956 (both at the school and university level), vast improvement with respect to the "extravagant" demands of society must be acknowledged. There are at present more than sufficient numbers of young people graduating from high school to pursue further mathematics training or to enter into various sorts of training to pursue the vastly increased number of careers that use mathematics. (We find it curious that this is essentially never mentioned in the hue and cry about the supposed disasters of the era of "the new mathematics.")

There remain the "modest" demands of society. That is, are most people able to handle numbers and computation in ways that enable them to copy with the everyday (and pervasive) uses of number in society?

Most of these uses involve measure or calculation or interpretation of numerical data; for the purposes of this paper we focus only on calculation. But even discussion of such a narrow part of the average person's "need" for mathematics must begin by observing that there is a fundamental confusion in most such discussions between (1) ability to calculate accurately and (2) ability to use those calculations. That these are different issues is indicated by the results of the 1972-73 National Assessment of Mathematics. For example, 92% of 17-year olds and 86% of adults could correctly add \$3.06 + \$10.00 + \$9.14 + \$5.10; 78% and 74% respectively could do this subtraction problem: "If 23.1 is subtracted from 62.1, the result is ..."; yet only 1% and 16% respectively correctly did a moderately complicated checkbook balancing problem (NAEP Reports, 1975). Schools react to charges that students can't use mathematics by increasing skill and drill work aimed at calculation, but the probability is that Johnny can add--he just can't use addition.

Still, it could be argued that ability to compute is a necessary even if not a sufficient condition for meeting the modest demands of society. Whether things are better or worse with respect to simple computation ability than before the recent reform is presently a matter of considerable dispute. Although the debate has now nearly reached the point of a fruitless 'tis - 'taint argument it may be useful to review some of the evidence, and to do so from an historical perspective.

The expectation that all members of a society should have competence in arithmetic is a very recent phenomenon. Not until 1745 was arithmetic competence required for admission to a college (Yale) in the United States.

(Harvard did not require the four fundamental operations until 1807.) Only in this century have state laws required children to be in school long enough to learn arithmetic. (Jones and Coxford, 1970)

Arithmetic texts of the 19th century reflect the "faculty psychology" theory of the times and contain many problems we would consider today to be very difficult (e.g., multiplying two six-digit numbers) or intricate (e.g., calculating cube root by hand). The 20th century has seen a rather continuous movement away from including hard problems simply for the sake of having hard problems. As a result, at any given time people appear to feel that students are not as skillful as their parents. (See also Suydam, 1972.)

Boss reported in 1940 that median scores in grades 3 through 8 were lower in 1938 than 1916. She concluded

> "the lower scores of 1938 do not indicate lower efficiency in arithmetic, but rather that the curriculum has been deliberately simplified and that the pupils are younger and less selected than those of 1916. Not the lowering of standards as is often claimed, but the widening of opportunity for continued school attendance, produces the lower mass average."

Beckmann in a 1965 study of 1296 students in 42 Nebraska high schools found that

> "students entering the ninth grade in Nebraska are much more mathematically literate today than they were 15 years ago."

The questions used by Beckmann included both arithmetic and geometry. The mean score (out of 109 items) was 45.67 in 1951, 54.91 in 1965, indicating either that the test was difficult or that competence was not particularly high at either time.

Roderick compared Iowa students performance on the Iowa Tests of Basic Skills for the years 1936, 1951-55, 1965, and 1973. He reported:

> "The most dramatic differences observed were in the 1936 to 1973 comparisons on the various topics. The students in 1936 were superior at both grades six and eight in every area tested...
>
> In condensed form, this study presents evidence toward concluding that the modern mathematics curriculum and/or its implementation is seriously deficient and ineffective relative to most of the long-term and still-held curricular goals identified."

Our own assessment of the Roderick data indicates to us that most of the losses he identifies show up in the 1955 versus 1936 comparison, with quite mixed results from then on. So we question the strength of his conclusion.

Roderick's conclusions fit what seems to be prevailing public opinion. But in Ontario, where the "new math" controversy has been as heated as in the United States, a recent extensive study concluded

> "In contrast to the widely publicized opinions of critics, this study clearly indicates that elementary school students in 1974 are just as capable of arithmetic computation as their age mates of 10 years ago." (OISE, 1975)

A carefully written article summarizing the National Assessment data on computational skills asked:

> "What do these NAEP data say about the effect of "new mathematics" programs? After all, the 13-year-olds and 17-year-olds could have been taught throughout their school experience in new mathematics classrooms. If the critics of the "new mathematics" were correct, the computational skills of these age groups would be very low. In fact the data show that 13-year-olds

> can do about as well as adults on most computational tasks, and 17-year-olds can do better. Remember, the adult population would not have been affected by exposure to new mathematics programs."
> (Carpenter, et. al., 1975)

We already noted that calculation skill and ability to use mathematics are different issues. On the NAEP tests, only about one half of the 17-year-olds and adults (aged 26-35) could solve typical consumer problems, with the adults consistently performing higher. The interpreters concluded that this might be due to either increased life experience of adults with such problems or due to decreased emphasis on such problems in newer curricula.

What can we conclude? Although historically, arithmetic skills have been the foundation of the elementary school curriculum, the performance on these skills has never been as good as some would want. Performance on tests on computational skills has provided continual fuel to criticize the curriculum. Whatever can be said about the present curriculum, research studies show no consistent pattern of declining achievement over the last 20 years. In any case, such concern may be misdirected, as we have noted, because even without the advent of calculators, the ability to use computation, not computational skills as such, is probably the central issue.

In this historical framework, calculators create a novel situation. For the first time, the need for computational skills is brought into question. (Recall that the newer mathematics programs, while perhaps spending less time on computation itself, proceeded under a belief that increased understanding of the underlying theory would cause greater proficiency in skills. Never was the need for skills seriously questioned.) There is no more fundamental tenet of the mathematics curriculum than the necessity for teaching arithmetic skills. The fact that calculators force one to question this tenet may account for the intense public interest and for the extensive media coverage noted above. The potential for wasted, misdirected, or plain wrongminded effort in accommodating to calculators is so great that it clearly warrants considerable and thoughtful consideration, especially in these early stages. Since any consideration must begin with existing school practices with respect to the teaching of calculation, we turn to that next.

C. The canonical calculation curriculum: numbers and their processing in grades K-6

Since the calculator principally calculates, any assessment of its role in the curriculum is bound to focus on how it might interact with that part of the curriculum that deals with numbers and their processing. Hence it seems useful to attempt an assessment of typical patterns of schooling in those areas.

It is the impression of many of us that "the calculation curriculum" and "the mathematics curriculum" for grades K-6 are essentially two names for the same thing--that is, concern with calculation dominates those years to a very large extent. Even topics like measure, geometry, and probability frequently appear to be in books mainly to provide another context for calculation. However that may be, calculation concerns are at least the largest single component of the curriculum in grades K-6. To examine that curriculum, we looked at widely used textbook series from major publishers, because in the great majority of classrooms such textbooks determine the pace, structure, and actual learning materials for the students. For that purpose we went through two textbook series year by year and page by page and tallied the main content of each page into various categories. Here we include comments mainly on the calculation content, which does indeed dominate these books. The first such book is by now an old standby and has been very widely used: Investigating School Mathematics, Addison Wesley Publishing Company, 1973 edition--referred to hereafter as AW. The second is a newer series that seems to be attracting a considerable following, Mathematics Around Us, Scott Foresman and Co., 1975 edition, hereafter referred to as SF.

Before dealing with the calculation content of these textbooks (summarized on page 15) a comment on the ways in which such textbooks contribute to methods of teaching in the classrooms they dominate may be in order. First, in spite of Piaget, and Dewey before him, there is little acknowledgment in the textbooks themselves that the early (say grades 1-3) number experience of children might profitably be based on work with concrete materials such as counters or colored rods. To be sure, the books for these early years are lavishly illustrated with pictures of many objects and diagrams of concrete situations (e.g., for addition: pictures of animals in a pasture with others coming to join them) but it seems pretty clear that looking at pictures can't qualify as a concrete operation. Teacher manuals for both books (but especially for SF) suggest activities with concrete materials but our overwhelming impression from observing classrooms is that these suggestions are rarely followed. This overweaning dependence on symbol manipulation of one sort or another as the nearly exclusive diet for early work with numbers contrasts with what nearly all of the best informed people in our field say would be more appropriate. It is easy to imagine that it accounts for the poor intuition for and understanding of number work on the part of many people that has characterized the results of school mathematics education for a very long time.

To turn now to calculation as taught in these books some general impressions from our tally of calculation content in SF and AW can be summarized up as follows:

(1) It is quite uncommon in these books for significant preparatory work for a given topic to appear at grade levels before the main introduction of the topic into the sequence. For example, there are essentially no fractions treated before grade 4, and no operations even with simple fractions before grade 5. This lack of gradual buildup may be another result of the dependence of books on symbols, since the appropriate preparatory experiences should very likely be quite concrete in nature.

(2) The general pattern of developments in both books as revealed by the page count is very similar although the books themselves seem quite dissimilar. An exception is that the SF book stresses decimal work on more pages (by a factor of 3 or so) and usually a grade level earlier. (The SF also has a more "applied" flavor throughout as context for the computational work.)

(3) The general pattern is introduction of number facts or new notation at one grade level with no prior work, then development of algorithms and the like with increasingly complex examples at the next two grade levels, with maintenance thereafter by review, use in story problems, etc.

As to the actual calculation content, Table 2 and Table 3 summarize the tally of pages in two ways. The relative emphasis in each book varies somewhat, but the mean captures very well a general impression of the sequence and emphasis in these books. Table 2 gives those means. Table 3 is a very much simplified version of Table 2; a check (✓) indicates a beginning on a topic, often near the end of the textbook for that year; a plus sign (+) indicates main emphasis; a zero (0) indicates little or no attention to the topic at that grade level; the letter (m) indicates at most skill maintenance exercises.

Table 2 Number of Pages per Calculation Topic Averaged from Two Textbook Series

		Grade Level					
		1	2	3	4	5	6
Whole Number Addition	Addition Facts	104	80	26	10	10	8
	No "carry"	10	15	10	0	0	0
	Isolated "carry"	0	28	26	12	0	0
	Arbitrary sums	0	0	10	15	20	17
Whole Number Subtraction	Subtraction Facts	88	58	28	9	7	7
	Single regrouping	0	27	24	7	0	0
	Arbitrary a-b ($b \leq a$)	0	0	14	18	22	17
Whole Number Multiplication	Multiplication Facts	0	17	35	27	9	9
	x 10, 100, 1000	0	0	9	15	9	7
	Single digit 2nd factor	0	0	18	16	6	2
	Arbitrary $a \cdot b$	0	0	0	17	25	20
Whole Number Division	Division Facts	0	1	28	19	13	7
	Single digit divisor	0	0	6	16	15	0
	Arbitrary $a \div b$	0	0	4	20	28	34
(Rational) Fractions	Meaning; equivalence	0	0	0	40	45	21
	$\frac{a}{b} + \frac{c}{d}$	0	0	0	0	28	23
	$\frac{a}{b} - \frac{c}{d}$	0	0	0	0	27	18
	$\frac{a}{b} \cdot \frac{c}{d}$	0	0	0	0	13	22
	$\frac{a}{b} \div \frac{c}{d}$	0	0	0	0	2	12
(Finite) Decimals	Decimals as						
	Money: add, subtract	0	0	5	6	0	0
	Meaning of decimals	0	0	0	0	9	4
	Add or Subtract	0	0	0	4	17	18
	Multiplication	0	0	0	0	11	20
	Division	0	0	0	0	0	16

Table 3 Grade Level of Introduction and Emphasis of Computation Topics in Two Textbook Series

		Grade Level					
		1	2	3	4	5	6
Whole Number +, -	Facts	+	+	m	m	m	m
	Easy	0	+	+	m*	m	m
	Mature	0	0	✓	+	m	m
Whole Number x	Facts	0	✓	+	+	m	m
	Mature	0	0	0	✓	+	m
Whole Number ÷	Facts	0	0	+	+	m	m
	Long Division	0	0	0	✓	+	+
$\frac{a}{b}$	Meaning	0	0	0	+	+	m
	+, -	0	0	0	0	+	+
	x	0	0	0	0	✓	+
	÷	0	0	0	0	0	✓
Finite Decimals	Money: +, -	0	0	✓	✓	m	m
	Meaning	0	0	0	0	✓	m
	+, -	0	0	0	✓	+	+
	x	0	0	0	0	✓	+
	÷	0	0	0	0	0	✓

*The "m" designation, meaning "maintenance" of skills, can mean work in problem sets or implicit in other operations. In that case, pages devoted to the subject don't appear, and hence are tallied as "0" in Table 2.

The main thing that strikes us in this tally is the extent to which new symbolic work of a fairly complicated sort is piled on in fifth and sixth grades. The first four years consists of fairly leisurely work with counting and whole numbers: the meanings of the whole number operations, extensive algorithmic work with addition and subtraction, and some with multiplication. The final two years of the K-6 school sequence have a heavy load of precisely those topics well known to be troublesome to students: long division of whole numbers; common denominators; division of fractions; and virtually all work with decimals beyond addition and subtraction. Both the amount and complexity of symbolic calculation is very substantially escalated in these two years and it is not surprising that this proves to be very difficult for very many students.*

This curriculum sequence may help to explain the gut reaction of so many teachers to the use of calculators in schools: "Fine, but not until after at least sixth grade." More is at stake in that than merely the feeling that one should learn to compute "by hand" before getting a machine to do it. Even moderate use of calculators before about fifth grade would upset this standard sequence radically by bringing a number of things into the school situation before their existence is acknowledged by the canonical curriculum. For example, calculators introduce multiplication of anything by anything; division of anything by anything; division that usually leads to decimals; decimals more generally, and not merely decimals for money. That is, if one accepts that the present sequence of calculation development in schools is necessary and logical, then one must view the calculator as disruptive.

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*According to followers of Piaget, many children in fifth and sixth grades may still be working at least in part in a concrete operations stage. But most of the work just referred to is heavily symbolic, and "understanding" what is going on requires fairly ornate formal arguments. This is especially so in the likely absence of work with concrete embodiments of the operations.

We believe that better sequences could easily be devised and that the calculator could play a fruitful role in these sequences. At a minimum, it would not be difficult to argue that the canonical curriculum has very serious weaknesses and is overdue for reconsideration.

For example, the canonical curriculum of the primary school years is surely quite weak. Too little is known about how young children learn best, but even what is known is often ignored. One signal of this is the almost universal failure to include even the simplest of concrete manipulations in early number work. First grade workbooks usually consist of page after page of equations with single-digit addition and subtraction accompanied at most by pictures of corresponding groups of objects and most children apparently do spend their mathematics learning time filling in the blanks in those workbooks. Suitable concrete work should enable earlier introduction of non-negative decimals (as an extension of place value) and, if so, addition and subtraction of simple decimals could probably be included not much later than the addition and subtraction of whole numbers.

One example of curriculum materials that attempt (only) the rudimentary reform of including considerable concrete work without changing the content and grade level sequence of the canonical curriculum is Developing Mathematical Processes. At present it has not gained wide acceptance. It may be that it attempts so much by way of variety of concrete work (admirable to be sure) and demands such special sets of equipment that it is seen as impractical for school use.

A more promising approach to the early school calculation curriculum is the fairly radical departure from it implicit in the Comprehensive School Mathematics Program (CSMP). The program includes considerable concrete work. But more than that, decimals and fractions as well as operations using them are included from first grade on. Furthermore, the operations in the primary grades are not restricted to addition and subtraction, contrary to the practice in both the canonical curriculum and the DMP variant. This is accomplished with concrete devices that allow for fairly ornate algorithmic processing. (The "Papy minicomputer" is the principal one used in CSMP, but other possibilities also exist.) These concrete calculators do more than accomplish the calculation work. They also embody important features of standard algorithms--for example, "carrying" and "borrowing" in addition and subtraction of whole numbers; power of ten shift rules for multiplication and division. To some, this might suggest that the variants suggested by CSMP are ideally suited to a calculator based curriculum, with the more efficient electronic calculator replacing the likes of the concrete minicomputer. But we think this suggestion would be unwise--unlike the concrete calculators an electronic calculator obscures the process by which an answer is arrived at. But it would be an interesting exercise to consider the calculator as a teaching aid in the primary grades of such a curriculum, as an occasional alternate to the concrete algorithmic processing.

In Part III we will consider in greater detail some possible suggestions for the role calculators might play in the elementary school calculation curriculum.

Part II

Changes in Curricular Emphases Suggested by Calculators

In this part of our essay, we turn from our general considerations of the present situation to more specific suggestions regarding the future. Finally, in Part III we become even more detailed in our discussion of calculators and the curriculum.

A. General Comments

In this section, we suppose that a curriculum is to be planned which postulates the availability of calculators both in and out of the classroom. This assumption forces questions of selection and ordering of computational skills (e.g., division of decimals) and of algorithms (e.g., some way of getting the correct quotient).

One easily-stated option which presents itself is that skills should be limited to those which are presently memorized and that all algorithms should be described in terms of pushing buttons on the calculator. This option has been supported by some proponents of the use of calculators and is often found as a "straw man" set up by opponents of calculator use in classrooms. There has not been enough time, though there might be experiments underway, to determine what would be the result of the replacement of all paper-and-pencil algorithms by calculator algorithms. We can only comment from our own experience.

While it is true that generations of students have done reams of calculations without profiting from this experience, it is also probably true that, among successful students and users of mathematics, some considerable part of their intuition may have developed as a result of this work. Thus we believe that to discard paper-and-pencil calculations in their entirety would be unwise, even if these are little needed in the everyday world.

On the other hand, the historical trend away from complex manipulations seems to us to have had foresight. Drilling students on such problems as 346.254 X 18.97 would seem to have little use except as a game. So we believe that to keep all of the algorithms with the degree of proficiency which is presently expected, and thus to ignore the existence of calculators, would also be unwise.

This leads us to the following summary view of the place of skills and algorithms in a calculator-conscious curriculum:

1. Students still must be required to have the fundamental addition, multiplication, and subtraction facts available for immediate recall.
2. Some work is needed with paper-and-pencil algorithms (though not necessarily the ones now in common use) for each of the four fundamental operations.
3. Younger students should have proficiency with the simplest types of computation in each of the four fundamental operations.

However, the rationale for having these skills must be modified. We presently teach all arithmetic skills as if accuracy in computation is the only goal. With a calculator present, accuracy is not a problem. The strongest reason for having competence with simpler problems is to have the ability to estimate the answers to harder problems. The time presently spent building up accuracy on quite difficult problems should be decreased and the resulting time should be spent on estimation and application.

4. The use of simple computations to help estimate answers to more complicated computations must be given strong emphasis.

Some have written about the use of calculators to check paper-and-pencil calculation. There are times when this might be an appropriate use of calculators (e.g., for self-checking and immediate feedback) but overuse of this device seems unwise. For suppose that a student labors over complicated arithemtic exercises and a calculator is routinely used to check the student's work. The student may rightfully wonder why he or she has to learn the skill if the calculator can do the problem more quickly and accurately.

The reverse process, however, is most appropriate especially with respect to mental or paper-and-pencil estimation of answers either before or after a problem has been worked on a calculator. Detailed paper-and-pencil calculation as a method of checking for operator or machine error might sometimes be useful, but could easily be overdone.

We feel that these summary comments are helpful in guiding one's thinking about the roles calculators might play in the curriculum, but such comments require elaboration to be useful to curriculum designers. So we now turn to a discussion of specific skills and algorithms.

B. Specific Comments with respect to the K-6 Curriculum

The following skills are necessary for the most rudimentary kinds of estimation. Even were calculators to be universal, their importance would remain.

- addition of whole numbers (including memorization of traditional addition facts)
- subtraction of whole numbers (including memorization of traditional subtraction facts)
- multiplication by 1- or 2-digit numbers (including memorization of traditional multiplication facts)
- division by 1- or 2-digit numbers
- small integral powers of positive integers
- multiplication and division by 10, 100, 1000, etc., .1, .01, .001, etc.

The student needs to learn a paper-and-pencil algorithm for each of these skills. We do not have enough knowledge about algorithms to determine whether the conventional algorithms are the best available.

Simultaneous with the availability and widespread use of hand calculators has been the realization of the advantages of and legislation supporting the conversion to the metric system. Pedagogically, the extension of base 10 notation to decimals is now thought more natural than the switch to fractions. Any of these developments would be sufficient to argue for an earlier introduction of the decimal system (in short, decimals before fractions rather than after). Together they make an early introduction of decimals essential. The young student will see decimals on the calculator arising from even simple division problems; this may be the most appropriate time to introduce all kinds of decimals.

Thus many of the skills associated with decimals gain in importance. We have remarked earlier that estimation skills gain in importance. Specifically, the following skills need more emphasis in the curriculum, this emphasis to be given either by an earlier introduction of the skill or by greater attention to the skill once it is introduced.

rounding
estimation of answers to multi-digit problems by rounding to one or two significant figures
addition of decimals
subtraction of decimals
multiplication of decimals with two or less significant digits
division of decimals with a divisor of two or less significant digits
conversion of fractions to decimals
conversion of certain decimals (suggestion: decimals equivalent to k/n, with n between 2 and 11, k any whole number.)

There are certain skills in the canonical curriculum for which it is not clear whether the payoff has ever been worth the effort. Even if there did not exist one electronic calculator, we might have suggested that the teaching of these skills be delayed perhaps even as long as to the secondary school. Certainly there is no compelling reason for the student to learn every bit of arithmetic before the study of algebra. To be specific, we refer to the following skills with whole numbers and decimals:

addition of long columns of multi-digit numbers
long division where the divisor has more than two significant digits

multiplication where the multiplier has more than
two significant digits

Every problem of the above types is formidable to the extent that the accuracy of even good arithmeticians cannot be guaranteed. They are particularly the type of problem for which the calculator is a fast and reliable aid.

C. The Future of Fractions

Because it is clear that decimals require greater and earlier attention in order to exploit calculators, there is a body of opinion which asserts that fractions will lose much, if not all, of their importance. To us this opinion is often based upon a further assumption, namely that the total amount of time devoted to fractions and/or decimals should remain relatively constant. We do not necessarily subscribe to that second assumption, and so we do not believe that increasing work with one requires a decrease of work with the other. More penetrating analysis is needed.

Fraction notation is associated with every division problem, expresses ratios (e.g., probabilities, scales), and is found in geometric proportions and many formulas. For example, the change of mass with velocity is represented by the formula

$$m_o = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}}$$

One would have to have an ample supply of negative powers in order to write the formula without fractions.

$$m_o = m(\sqrt{1 - v^2c^{-2}})^{-1}$$

(While the use of negative powers would be an interesting notation to foster and perhaps to study, it does not seem feasible given current practice.) The solution to as simple and fundamental an equation as $ax = b$ is usually written as the fraction $\frac{b}{a}$. Thus it would seem to be disastrous not to give strong attention to the meanings of fractions and, concurrently, to the notion of equivalence and simplification of fractions.

With respect to operations with fractions, the situation is less clear. First, paper-and-pencil work with fractions is not easily converted to work on the calculator. Unless one is working with a calculator which utilizes reverse Polish notation or multiple parentheses, the problem $\frac{2}{3} + \frac{1}{4}$ cannot be finished by pressing $2 \div 3 + 1 \div 4$ on the calculator. Even in reverse Polish notation, the answer will not appear as a fraction. This suggests that the symbolic algorithm $a/b + c/d = (ad + bc)/(bd)$ should be taught, with the calculator (if needed) used only to calculate the numerator and denominator of the answer. (This suggestion was made by some before the advent of calculators due to the frustration students have learning the normal "least-common-denominator" algorithm for addition of fractions and the easier transfer to some work with fractions in algebra.)

In short, if operations with fractions as fractions are important, then having a calculator does not shorten what is to be learned. The calculator only shortens the arithmetic. Since traditional work with fractions has used small positive integers for the most part, the calculator is not particularly helpful.

But are the operations important? Addition and subtraction of fractions usually occur with measurements; if measurements are to be solely in decimals (as is the case even now with money and temperature), then time spent on these skills should be decreased. For the few places that addition or subtraction of fractions would remain in use (the stock market, perhaps), the skill could be learned closer to the actual application.

Multiplication and division of fractions occur with respect to ratios, rates, and proportions. For example, you may divide fractions if you know that your car gets $16\frac{1}{2}$ miles to the gallon on the average and you wish to know how much gas you have used if you have travelled 260 miles since the last fill-up (an actual problem experienced by this author within the last month - and the fraction is crucial when the gauge registers less than empty!). If a model is to be 3/8 actual size and you want to know how long certain dimensions will be in the model, then multiplication of fractions is appropriate. It is clearly possible to do all of these calculations with decimals. But 3/8 is sometimes easier to picture (what has length 3 in the model has length 8 in the original) than .375. If the calculations are done in decimals, will understanding of the situation be decreased? Or does that question assume student understanding which is often lacking and cannot be decreased!

The above arguments indicate three suggestions with respect to the teaching of fractions in a calculator-available curriculum:

1. Less emphasis should be placed upon arithmetic operations with fractions.
2. More emphasis should be placed upon symbolic manipulations with fractions.
3. Continued emphasis is needed on the meaning of fractions and work with equivalent fractions.

D. Arithmetic Skills Introduced in Grades 7-9

The arithmetic skills normally introduced in grades 7-9 are the following: work with percentages, calculation of square roots, the four fundamental operations with negative numbers, and occasionally scientific notation and/or work with negative exponents. In all cases, these skills are considered more difficult than those skills of the K-6 canonical curriculum and judging from the content of remedial texts, with the exception of work with percentages they are considered less important.

Work with percentages remains an important skill for the consumer to have. Decimals make it possible for such work to be given earlier. Though we would prefer that the "percent" key on many calculators be replaced by a key with a nonredundant function, the presence of this key could also motivate early introduction.

A similar early introduction would seem to be natural for negative numbers. The common saying "You can't take 5 from 3" was always silly but is not even applicable when calculators are available. A student who punches in a subtraction problem in reverse order will see negative numbers correctly appear. But any early introduction will be impossible unless it is coupled with the variety of common life situations in which negative numbers make it easier to operate: profit-loss in business; gain-loss in weight, football, or stocks; ahead-behind in bowling or golf; east-west or north-south movement; height above or below a fixed level; and so on. The general public, despite the use of negative numbers on television and in newspapers in situations like the above, still views negative numbers as artificial and lacking any meaning in the real world and for this reason may not support any curriculum in which all students receive early introduction of negative numbers.

Scientific notation has always been used to estimate very large or very small numbers. On some calculators, a knowledge of scientific notation is needed to interpret answers. If estimation is to be given the stronger role that it deserves, scientific notation as a tool in estimation and representation of numbers will increase in importance. This would force an earlier introduction of addition and subtraction of positive and negative integers as well as increase the already important role given to powers of ten.

Though there are reasons to introduce the meanings and interpretations of negative numbers earlier than in the present canonical curriculum (i.e., earlier than grade 7) and work with addition and subtraction of these numbers before grade 8, it does not seem necessary to multiply or divide with negative numbers any earlier than in current practice. We make this statement cognizant of applications of these operations (changes of direction and size as examples of multiplication; rates of loss or going back in time as applying division) but feel that there are more important prior curricular considerations.

Calculation of decimal approximations to square roots by either the Archimedean or "divide-and-average" algorithms is an anachronism. To most students these algorithms are of mysterious origin (even if the teacher takes the time to give their mathematical justifications) and they do not contribute to an understanding of what a square root is. The authors, neither of whom considers himself to be poor at computation, have never trusted their own work with these algorithms. Given calculators, the best algorithm would seem to be to test potential square roots or approximations to square roots by squaring these numbers. (We also believe that a square root key is more useful and more justified than a percent key on calculators, but on this point the public may have more influence than mathematics educators.)

E. Remediation in the Secondary School

Remedial work with arithmetic skills and algorithms characteristically takes place in "general mathematics" and, to a lesser extent, in "two-year algebra" courses taught in grades 9 and 10. Judging from available materials, in some of these courses there is little other content than this arithmetic, in others the arithmetic is placed in a "consumer" context, while in still others a potpourri of mathematical topics including computation is studied.

Here we address the question of the effect of calculators on the need for and treatment of calculation in these courses. Again we believe that the wisest strategy lies somewhere between the unqualified "Yes" (use calculators for everything) and "No" (don't use calculators at all) positions, but we are strong in our belief that on this Yes-No continuum the most appropriate use is closer to the "Yes" end.

Students in these courses characteristically are very weak in arithmetic skills. That is, they have not been able to assimilate many of the paper-and-pencil algorithms which they have presumably been taught in grades K-8. As a result, many of the patterns (and, as a consequence, the understandings) which more proficient students see in numbers have seldom been seen by these students. And the applications seem remote if not impossible to carry out. It does not seem to be a formidable task to prepare materials which could help these students to explore both patterns and applications.

Certainly, there is little reason to continue what have been ineffective procedures. Thus we recommend that only the following topics be covered without the presence of a calculator:

addition of whole numbers (including memorization of traditional addition facts)

subtraction of whole numbers (including memorization of traditional subtraction facts)

multiplication by 1- or 2-digit whole numbers (including memorization of traditional multiplication facts)

division by 1- or 2-digit numbers

small integral powers of positive integers

multiplication and division by 10, 100, 1000, etc., .1, .01, .001, etc.

work with fractions as suggested on page 28

meaning of decimals

All other arithmetic skills should be covered with a judicious balance of paper-and-pencil estimation and calculator calculation.

Opponents of calculator use in remedial courses give arguments which we feel are moral in nature. It is necessary to consider these arguments but such consideration would lead us away from the specifics which characterize this section. We have treated one such issue in Appendix C.

F. Summary

We suppose that a curriculum is to be planned which postulates the availability of calculators both in and out of the classroom. In the following table, we summarize the importance of various skills in such a curriculum as compared with the treatment in the canonical curriculum.

Fundamental Operations:	Addition	Subtraction	Multiplication	Division
Whole numbers:				
≤ 2 digits	0	0	0	0
> 2 digits	0 (columns -)	0	-	-
Decimals:				
≤ 2 sig. digits	+E	+E	+E	+E
> 2 sig. digits	0	0	-	-
Fractions:				
calculations	-	-	-	-
symbolic manip.	+	+	+	+
Estimation of answers:	+	+	+	+
Negative Numbers:	0E	0E	0	0

Conversions:	
fractions to decimals:	+E
decimals to fractions:	0 (certain ones +)
Percentages	0
Square Root Calculation	-
Scientific Notation	+E

Code: 0 = likely to remain equal in importance

+ = likely to increase in importance

- = likely to decrease in importance

E = likely to appear earlier

Part III

Notes for an Alternative Calculation Curriculum

A. General Comments

When one speaks of "the" mathematics curriculum, it is important to distinguish between (a) the "canonical" curriculum, as represented by current best-selling textbooks, (b) already existing newer curricula, such as those represented in the DMP or CSMP materials, and (c) a hypothetical or envisioned curriculum which exists only in someone's mind or in rough outline form. The uses one sees for calculators vary according to whether one's frame of reference is a curriculum of type (a), (b), or (c).

Let us review some observations made earlier. In the canonical curriculum, arithmetic skills have been taught as an end in themselves. Though the ability to apply these skills in the world is a universal goal, the materials do not reflect this goal and recent tests suggest that the skills are learned but not the application. The absence of consumer-oriented problems on some standardized tests for grades K-6 or even K-8 is a further indication of this skill orientation.

The canonical curriculum contains almost no work in estimation or approximation, only the most rudimentary applications, and little in the way of handling data. In classroom practice, there is virtually no concrete work, even in the primary grades. And as we have noted earlier, the numbers and operations used at a particular level tend to be strictly controlled, parcelled out slowly, and with little grounding at one level for what comes next.

Now let us suppose that we insert calculators into this canonical curriculum without making curricular changes. First of all, the entire testing framework is affected. Exactly those answers which students spend hours, days, and months learning how to find are those answers which the calculator computes quickly. A good teacher will devote time to explaining why an algorithm works, but a calculator hides the algorithm, and seems to make all that superfluous. Thus the calculator would seem to destroy current rationales for learning arithmetic skills.

Furthermore, what the calculator does well is not reflected in the canonical curriculum. The calculator allows for the handling of more complicated data. It makes trial and error methods feasible in many contexts. The calculator displays a huge variety of real numbers not usually encountered in the early grades. It is not until grade 7 at the earliest that the canonical curriculum involves all of the numbers and operations displayable on the simplest 4-function calculator. Also, exploiting the calculator requires some knowledge of estimates, knowledge which usually does not result from this curriculum.

It is thus our belief that the insertion of calculators into K-6 classrooms using most existing curricula is fraught with peril. There is the potential for abandonment of those skills which are necessary even given universal availability of calculators (see part II), there is a complete shakeup of the fundamental reasons for teaching mathematics in these grades without a conspicuous alternative represented by the materials being used, and the unique features and abilities of calculators introduce concepts and numbers into the classroom which are not reflected in the materials. And in a curriculum that already has virtually no manipulation of concrete objects, the calculator removes most of the manipulations with numerals.

Both the DMP and CSMP curricula seem to be more suited to the appearance of calculators. Each is more attached to the real world. Each utilizes concrete materials. Each does work with estimation. The CSMP curriculum might be the better suited of the two, due to an earlier introduction of many types of numbers.

But, to our knowledge, neither curriculum allows for calculators or gives special activities utilizing unique aspects of calculators. (One could not expect the designers to have foreseen the appearance of these devices so quickly.) In each, the calculator could be an adjunct with fewer, but still with existing, threatening aspects.

Thus it seems that if calculators are to be used in the elementary grades, alternative curricula must be designed with calculators in mind. We believe that it is possible not only to make room for fruitful use of calculators but also to alleviate some of the weaknesses in the canonical curriculum. As something of a partial existence proof of this possibility, the pages that follow include pieces of a rough outline for one such curriculum alternative for the elementary grades. The main features of this alternative are greatly enriched content in the early grades (as in CSMP), work with concrete materials, and, of course, use of calculators.

In considering the material that follows we ask the reader to keep in mind the fragmentary and tentative nature of the suggestions made. With respect to content we are concerned only with number and calculation and even in that we are not exhaustive. With respect to sequence, the (partial) outline is at most only suggestive. With respect to work with concrete materials, only a small sample of the possibilities is listed. (A more complete outline would carefully assess what is by now a rich field in order to specify what would best embody each idea, and perhaps also what would mesh best with the presence of calculators.) With respect to calculators, most of the exercises suggested have emerged from preliminary and quite informal trials of calculators by twenty teachers or so; they clearly do not exhaust the possibilities. Furthermore the outline is uneven, with much more detail for early work than for later work. This is mainly in response to the widespread impression that calculators have potential only (or at most) for work in the upper elementary grades and beyond. That is, we outlined the early grades in more (but still far from sufficient) detail because that is where there appears to be most skepticism both with respect to richer content and with respect to fruitful teaching uses of calculators.

Finally, the reader should know that what we regard as perhaps the most promising teaching use of calculators is virtually omitted from the material that follows. We refer to the possible role of calculators in helping youngsters make sense out of numerical data. We assume that at almost any level, problems beyond the paper-and-pencil calculation power at that level can come from contexts that youngsters nevertheless understand and find interesting. Assuming the youngsters can understand both the data and results from processing that data, the calculator can give them power to do the processing. In further development of any such curriculum as is suggested by the material that follows, we believe these possibilities should be fully developed. Examples could come from everyday experience, from many excellent new elementary science programs, from USMES-type challenges, from various sorts of projects including projects for social studies or language arts, from Nuffield-type activities, and so on.

To sum up, the pages that follow include merely some first draft fragments for what might profitably be developed into an outline for a full-scale alternative to present practice. No one is more aware than we that it is not yet a fully developed alternative. But we think this material may suggest that serious efforts to find a useful place for calculators in teaching elementary school mathematics (including the primary grades) could be rewarding. If so, the modest objectives of this fragmentary listing have been achieved.

B. Some possible activities for Phase I of a Calculation Curriculum: Introducing the Child to Arithmetic

In the first phase of the child's experience with arithmetic, each concept is introduced through actual manipulation of concrete objects. Inputs to and results from the concrete manipulations are to be recorded on paper in order to establish links among the concrete, the verbal, and the symbolic, and also to keep track of the processing. Because the calculator disguises the processing which it does, it is particularly important that the same kind of recording be used with calculator inputs and results.

Keep in mind the general caveats mentioned in the general comments above:

(a) Only calculation content is dealt with here; any reasonable curriculum for elementary school mathematics must include much which is not calculation.

(b) The outline is not intended to be exhaustive in any respect; we know that some important content may be neglected; many other useful concrete activities exist; there are many other ways to exploit calculators to increase understanding of arithmetic.

(c) Understanding of arithmetic can probably be reinforced in many ways by using calculators to help make sense of numerical information that comes up in school exercises or that youngsters generate or bring in, but we list few of these possibilities.

Some Skills or Concepts	Some Sample Concrete Embodiments	Some Sample Calculator Uses
(a) Whole Numbers as Counts		
*Count by 1's, say to 126 *Count by 10's, say to 500 *Start at n, count on; count back *Recording of results of a counting process, say up to 126 *Read a two-digit numeral *Count out a set of n objects, n reasonable	*Attach verbal counts to many sets of objects *Count out sticks, bundle by 10's Count by 10's with 10-bundles *Embody two-digit numbers with sticks and bundles; count on (continue bundling) and back (unbundle) *Form n; predict and act out (((n + 10)+ 10)+...); ((n - 10)- ...) *With reversible grocery counter, count on and back by 1's; by 10's	*Set constant so that repeated button pushing counts by 1; child counts verbally in pace with display *Similarly, count by 10's *Display number, then use constant to count on and back by 1's *Display n; use constant to count on and back by 10's; ask for prediction before pressing button *One child displays number he can read; challenges partner to read it *Children challenged to display biggest, smallest 1- or 2-digit numbers; disputes settled with concrete embodiments *One child says a number; partner displays it
(b) Whole Numbers as Measures		
	*Usual play and preliminary work with colored rods (e.g., Cuisenaire rods) augmented with clear acetate 1 x 1 cm squares to represent 0 *Measure lengths with rods; express results with 10-rods and at most one other *Use meter stick to measure; record to nearest cm	

Some Skills or Concepts	Some Sample Concrete Embodiments	Some Sample Calculator Uses
(b) Whole Numbers as Measures (continued)		
	*Children weigh themselves on bathroom scales (kg); weigh other large objects (kg) and small objects (g); record results *Water and sand table play with volumes--how many of these in this?	
(c) Meanings of addition of whole numbers		
*Counting—i.e., combining *Measuring--i.e., joining	*Combining sets of objects *Usual work with "trains" of rods; some of this with rods on number line *Add by counting on with grocery counter *Simple two-digit addition with sticks and bundles *"Add" using volumes, weights, etc.	*Get a + b results from games, data, a classroom store, other problems of interest to students, etc; occasionally use data beyond that which is easily acted out concretely *Explore patterns such as "adding 10," "adding 2," "adding 9," to arbitrary x *Fill in table of basic results with combined concrete and calculator work
(d) Meanings of subtraction		
*Counts or measures: take-away comparison *allow negative answers closely tied to real situations	*Use counters and rods in both take-away and comparison situations *Count back with grocery counter *Unbundle sticks for borrowing *Act out some money problems; classroom store is possible; also integers via losses, lending, etc.	*Explore patterns like (a+b)-b, (a+b)-a, a - a, etc.; explain the answers in terms of real situations *Explore a - b and b - a; compare a - b with b - a *Estimate answers to large problems; verify with calculator

Some Skills or Concepts	Some Sample Concrete Embodiments	Some Sample Calculator Uses
(e) Meanings of multiplication		
*Arrays *Repeated addition using counts and measures *Combinatorial problems *Count by n	*Count m groups of n counters *Count m X n array by n *Train of m n-rods; convert to 10a + b *Measure array of n-rods across by an m-rod *Act out combinations *Money for 10 X 1, 10 X 10, 10 X 100, etc.	*With a constant, count by n's; explore patterns *Calculate plausible problems with areas and arrays (e.g., how many tiles on the floor or ceiling) *Units in units: seconds per hour; minutes per day; meters in n kilometers, etc.
(f) Meanings of division		
*a things or something of size a split evenly b ways *How many b's in a *One-bth of a vs a÷b *Correspond with multiplication *Rate	*Share things evenly (e.g., cookies) *Forming groups for how many b's in a *a counters with b rows, "one for you and one for me," sometimes remainders *One-bth of a in money, sticks and bundles *Cut up ribbon of length a into b equal pieces; into pieces of length b	*Illustrative concrete problems; observe non-whole number quotients correspond to remainders. *Numeral n.d is between n and n+1; begin rounding and also lead-in to decimals *"Best buy" pricing
(g) Introduction to decimals		
*$n \leq n.d < n + 1$ *Decimal system and money; X 10 and $\times \frac{1}{10}$ or ÷ 10 *Record metric length measures with decimal notation; notion of "more accurate"	*Decimals from money transactions and from linear measurements *Calipers	*Grocery store problems; calculator as check-out machine *Explore accuracy needed in real situations

Some Skills or Concepts	Some Sample Concrete Embodiments	Some Sample Calculator Uses
(h) Meaning for fraction notation; equivalence of fractions		
*Parts of whole *Ratios * $\frac{a}{b} = a \div b$	* $\frac{1}{b}$, $\frac{a}{b}$ of a set of objects *Mark 24-cm "unit" on number line with rod trains ($\frac{1}{2}$'s, $\frac{1}{3}$'s, $\frac{1}{4}$'s, $\frac{1}{6}$'s, $\frac{1}{8}$'s, $\frac{1}{12}$'s, $\frac{1}{24}$'s); note many names for some points *Reprise concrete work for $a \div b$ *Paperfolding of unit strips $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{2}$ of $\frac{1}{2}$, etc.	*Check equivalence of fractions by division *Order fractions by division

C. Some Possible Activities for Phase 2 of the Calculation Curriculum: Building Up Skills and Concepts

Skill-building begins before Phase 1 ends and includes the basic addition, subtraction, and multiplication reflexes. During Phase 2 the algorithms for the fundamental operations are established and some paper-and-pencil competence and accuracy is expected.

The "concrete" work in this phase becomes more coded. For example, dowels of the same approximate size can replace 10-bundles (with only a few actual bundles left for regrouping). Work with counters can be done on a place-value sheet. This allows for work with decimals without needing physically smaller pieces for tenths, hundredths, etc.

The work of this phase is specified below in less detail than before, with the examples simply meant to suggest types of uses of concrete materials and of calculators. The main objectives here are to demonstrate the existence of fruitful work with these aids and to stimulate thought in these directions.

Some Skills or Concepts	Some Sample Concrete Embodiments	Some Sample Calculator Uses
(i) Addition, multiplication reflexes		
	*Check doubtful combinations with concrete work, but goal is reflex responses	*"Beat the calculator" to encourage mental processing *"Broken calculator" problems as practice--i.e., postulate only certain buttons work, ask for display of 1, 2, 3, ..., n
(j) Addition and subtraction skills		
*$a + b$ (a, b any integers, easy decimals) *$a - b$ (a, b any non-negative integers, easy decimals) *$a - b = c \Leftrightarrow c + b = a$ *$a + b + c + d$ for reasonable data	*"War" with black and red cards, the latter negative. *Chip computers to keep track of operations with multi-digit numbers *Meter sticks, tapes, trundle wheels for data gathering or picturing	*Break down calculations by parts--e.g., use calculator for one's column-record, then for 10's column, record, etc. (in subtraction, the negatives suggest borrowing) *Estimation of answers by using only first significant digit, or two significant digits
(k) Multiplication and division skills		
*$a \times b$ for up to 2-digit whole numbers on paper; arbitrary on calculators *$a \div b$ where b has up to 2-digits on paper, arbitrary on calculators *$a \div b = c \Leftrightarrow cb = a$	*Repeated addition (for single-digit b in $a \times b$) using dial-a-matic, grocery counter, chip computer, 10 blocks, or sticks and bundles *Exploration of 10-shifts with matereals, e.g., each X 10 shift is an exchange for a next higher value *Repeated subtraction (for single-digit b in $a \div b$) using above materials	*Exploration of X 10, X 100, ..., shifts both for whole numbers and decimals *Exploration of ÷ 10, ÷ 100, ..., shifts... Compare with X .1, X .01, ..., shifts *How do you multiply with multiplication key inoperative?--begin with repeated addition, move to combination of repeated addition and X10, X100, ... shifts to get to the usual algorithm

Some Skills or Concepts	Some Sample Concrete Embodiments	Some Sample Calculator Uses
(k) Multiplication and division skills (continued)		
	*Use dot arrays to exhibit 2-digit factors; group results by powers of 10 to suggest an algorithm	*Obtain partial products, then add *How do you divide with division key inoperative? (1) begin with repeated subtraction, move to combination of repeated subtraction and X10, X100, ..., shifts to show possible algorithms: (2) revert to multiplication and careful trial and error

D. Some possible activities for Phase 3 of the Calculation Curriculum: Applying Arithmetic Processes

This is a phase which begins before phase 2 has ended, with wisps as early as Grade 4, and continues through the remainder of the child's arithmetic experience. Here one takes advantage of the skills and concepts built up in Phases 1 and 2 in a variety of general ways: using the skills to estimate answers; using the concepts to develop shortcuts and applications of the arithmetic; using both to search for and utilize algorithms for specific purposes in the processing and understanding of data.

In the outline we have only listed content which is not found in the previous phases. Of course, we expect that concepts and skills from the other two phases will be elaborated upon, as indicated in the preceding paragraph. Also, we have not listed all the new content that might be included in this phase.

In this phase, one relies on number patterns more than concrete embodiments. This is not to imply the absence of concrete work, especially for new concepts, but the reliance on concrete work should be diminished. In the place of some of the concrete work should be increased work with actual data, always with attention to questions of "reasonableness" of answers.

Content	Illustrative calculator work
(ℓ) Percentages (Here the calculator enables consideration of data which would otherwise be formidable.)	*If calculator has a percent key, demonstrate its redundance--i.e., equivalence of algorithms *"Evenness" of percentages; i.e., calculate 5%, 10%, 15%, ... of same number, note pattern; repeat with 95%, 100%, 105%, ... *Percentages from large samples--e.g., use actual data to estimate the percentage of people in the U.S. who live East of the Mississippi; discuss significant digits *Discounts on actual prices; 20% followed by 10%, etc.
(m) Unary operations: square root "greatest integer less than" and other roundings (Here the imprecision of the calculator fits well the student's view of square roots as being inexact.)	*Estimation of square roots by trial and error squares of numbers; algorithm for getting best possible calculator estimate *Analysis of calculator rounding; e.g., how does 1 ÷ 3 on the calculator compare with $\frac{1}{3}$. *Error compounding in multiplication or division; e.g., compare 2.49 X 3.49 with 2 X 3
(n) Powering (a^b where b is an integer, a arbitrary) (Here the calculator enables one to do calculations which could not otherwise be considered.)	*Calculation of powers of integers; patterns of final digits; verification of laws of exponents by calculator *Calculation of compound interest; e.g., consider how long it takes money to double at various rates *Comparison of exponential growth to linear growth *x, x^2, x^3; e.g., in connection with linear, area, volume, relationships *Demonstrations that $x^{n+1} < x^n$, and $x^n \to 0$ if $0 < x < 1$; applications to probabilities of independent events
(o) Scientific notation (Some calculators will force a need to consider this content, particularly in the context of powering.)	* Motivated by need to write very large or very small numbers on an 8-digit keyboard *Calculator for patterns in $10^m \cdot 10^n$ and $10^m \div 10^n$ *Repeated division to show negative exponents

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APPENDICES

Appendix A: Some Frequently Asked Questions about Using Calculators in Schools and some Tentative Answers based on Informal Trials*

It cannot be too strongly emphasized that these comments are based on quite informal trials and feedback from only about twenty teachers. Consideration of any particular question may depend on feedback from only one or two teachers. That understood, it still seems useful to try for first approximation comments on a variety of questions. Roughly speaking, questions about learning possibilities are at the head of the list, with questions that touch on administrative matters at the end of the list.

1. Is explicit instruction in using the calculator necessary?

Based on our trials we believe that at least from fifth grade on children learn to use a calculator very quickly (usually within an hour) with at most a worksheet that makes them confront various possibilities. They tend to learn to do the things they know something about; e.g., if they are unfamiliar with division they ignore that key. They learn both from the machine and each other; discoveries about shortcuts and particular quirks are quickly shared. (Despite this, there are already advertisements--e.g. in Learning magazine--for filmstrips on how to use a calculator!)

Certain concealed features, such as automatic constants or repeating operation keys sometimes need to be pointed out but correct use of them is quickly mastered once attention is drawn to them.

Some teachers who used the calculators in grades 1-4 taught several students explicitly and these students taught others. But self teaching was effective in the Page first grade and Garmony third grade groups.

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*This is part of Appendix 1 to Bell, M.S., Explorations into Ways of Improving the Elementary Mathematics Learning Experience, a report on a project supported by NSF grant PES 74-18938. The references herein to specific teachers or grade levels relate to anecdotal reports on classroom trials of calculators that are included in that same appendix, but which are not included here.

2. Do children "naturally" detect errors; that is, do they reject clearly unreasonable answers?

On this question we have mixed reports; some explicit research would be profitable. Tentatively we feel that older children who have good "number sense" can carry this over readily to calculator work but do not do so automatically (see Schaeffer report). Similarly, for younger students, some but not all of Page's first graders were uncomfortable about wrong answers and asked the teacher to look at them. Some upper grade students doing worksheets with division by .1, .01, etc., claimed the calculator was broken because it gave bigger answers--that is, they noticed what _seemed_ to them to be a wrong result. But many youngsters in all grades seem accustomed to accept whatever "answer" a calculation leads them to and this carries over to calculators. Thus, for example, many children unfamiliar with decimals ignore the decimal point altogether in writing answers, no matter how ludicrous the result. As to how many significant figures to keep, nearly all children write whatever the calculator says--and often wish aloud there were _more_ than the usual eight digits.

It seems to us that school neglect in teaching of approximation skills and of good sense about significant figures is plainly revealed when kids use calculators. Such things simply _must_ be more emphasized if calculators are used in schools, but they should be more emphasized in any case.

3. Has the calculator any potential for diagnosing gaps in understanding of content?

We think there is considerable potential for this though none of our trials were directed to that end. For example, when using calculators with eighth graders we learned _very_ quickly who does and who does not understand what decimals mean, even for youngsters who could do certain calculations using decimal numbers. As another example, in a certain sixth grade class, the problem 38 ÷ 144 was answered (on a six-digit machine) as 3.78947 by six students, 378947 by two, 26388 by four, and .26388 (the correct answer) by only two students. Those various responses give very clear signals for follow-up. Using the same problem with about 75 eighth graders gave similarly clear clues to difficulties (but far more students did it correctly, as one would expect). Also, every problem-solving use reveals only a minority that understand about significant figures in measure and calculation.

We believe the diagnostic possibilities are considerable and further work on them would be warranted.

4. Do children become curious about functions on the machine that are unfamiliar to them?

As far as our trials go, this is an open question. Third graders asked for more information about multiplication, but they already knew about multiplication in simple cases. These same youngsters asked about the meaning of decimals and settled (for the time being) for a simple "whole number plus a little more" sort of answer. They did not ask about division and ignored the division key. Similar results were found with first graders. All the sixth graders discussed in #3 above did the division problem demanded of them (they might have ignored division otherwise), but most did it wrong and no one, apparently, was moved to ask about it. Tentatively, the obvious answer seems probable: valid discoveries from completely unguided exploration of unfamiliar keys is unlikely but there may be considerable potential for guided exploration of partly familiar things. The unknown keys seem not to be distracting, so it at least may do no harm to keep further mathematical possibilities in the environment (by way of extra machine keys) and for certain youngsters and certain teachers some nice explorations might result. That possibility needs to be balanced against extra cost. I would like it if at least $\sqrt{\ }$ were on nearly all machines, perhaps instead of the usual % key.

5. Are children interested in the calculators and if so, does the interest last over an extended period?

There is invariably very high initial interest. From our trials, we find that high interest persists over a long time period provided students are given interesting things to do with the machines; indeed they demand to have things to do with them. Nearly every teacher in these trials commented on how motivating the calculators appear to be to kids and not a few teachers have said that "discipline problems" virtually disappear when the machines are used, even in quite difficult situations. The main problem, of course, is that few school mathematics materials have really interesting problems in them that exploit the power given youngsters by the calculators--a situation that most certainly should be remedied, and the sooner the better!

It could happen, of course, that interest in the calculators will wane as they become a very familiar feature of our society. But we have seen no evidence that "familiarity breeds contempt" either in these trials or for individual youngsters of our acquaintance who have now had calculators for several years.

6. Do children become dependent on the calculators? Does it matter?

Without discounting the possibility of overdependence as a long range effect or a result of unwise pedagogy it is clear from our trials to date that this is not a significant problem. Children at first do everything in sight on the machine but in all our trials they rather quickly gained good judgment about doing easy things in their head (whenever they could) and using the machine at most for things they would otherwise have done by pencil and paper algorithms. It is clear that many critics of the use of calculators in schools would regard any use in place of pencil and paper as leading to overdependence but there appear to be easy safeguards. For example, most teachers in these trials periodically demanded paper-pencil computation even with calculators present and the students seemed to go along with this without resentment.

It is unclear to us how much to worry about possible increased dependence on calculators. "The batteries may run down" as the main argument for no de-emphasis at all of hand calculation seems somewhat silly. More troublesome is the fact that we know very little about how children learn mathematical concepts, or even what they learn from the usual sequence of experiences. That being so, it would be unwise to discount the possibility that algorithmic manipulation of numbers as such contributes to the learning of important mathematical concepts. It is easy to imagine that the very intricacy of the manipulations plus the patterns and rules that make them work may sometimes result in important, even if unspecified, learnings. The existence of calculators suggests ways of inquiring into the issues here and this is another area where research should be fruitful (with, of course, appropriate safeguards). In the meantime, a conservative (but not immovable) posture toward the learning of calculation in schools seems warranted.

"There can be no doubt of one thing: good "reflexes" with respect to basic multiplication and addition results ("learning and tables") remain very essential. With or without a calculator it is crippling not to have such reflexes. We feel the same about a very sure feeling for effects of multiplication and division by powers of ten--a feeling that far too many people now fail to acquire.

7. Is choice of machine configuration an important issue?

The first thing to be said is that in our trials children adapted to a variety of machines without difficulty and seemed to be able to switch from one machine to another without confusion. Thus it seems safe to say that at least for older children, any machine that gives reliable results could be used. But there still remain intriguing unanswered questions that merit investigation, and this is especially true for use of calculators with children in the primary grades.

In addition to power source (rechargeable battery, ordinary battery, AC adaptor) three aspects of configuration need consideration:

a) display: left to right entry or right to left entry; scientific notation or not; number of digits and possible choice of that; size and color of display; etc.

b) keyboard: which functions and number of functions; change sign keys; memories; constant--automatic or keyed; repeating keys; multipurpose keys; etc.

c) type of logic: algebraic (equation) logic, arithmetic logic as on some inexpensive machines; reverse Polish notation (RPN), with a "stack" that sets it markedly apart from the similar arithmetic logic.

These various aspects are combined in a bewildering array of choices in the calculator marketplace. But by now an inexpensive (under $10) consumer oriented machine has emerged that is virtually identical among major distributors: non-rechargeable battery with adaptor available; eight digit display with entry from the left; the four standard operations and a fairly versatile percent key; automatic constant which makes powers, reciprocals, repeated products, sums, and differences available by repeated punching of operation keys; no multiple use keys, except possibly C/CE. A few more dollars buys very similar models with memories, and a few more dollars buys similar models with square roots, reciprocals, and, sometimes, scientific notation. Beyond that considerable variety remains the rule.

A number of people have by now delivered themselves in print of opinions about what sort of machine should be purchased for school use. These opinions are remarkably similar to each other: algebraic logic, keyed or automatic constants, rechargeable batteries, and no multiple use buttons. The trouble is that this standard advice is not based on any evidence about what may work best pedagogically. Over the long run such issues deserve investigation. Findings from investigations can influence design since the schools market will presumably be important enough that calculators to given specifications can be made available.

The possible pedagogical ramifications of various combinations of display, keyboard, and machine logic are far from clear at present but such issues certainly exist and they may be non-trivial, especially for calculators used with younger children. Here are a few such issues and questions suggested by our eighteen months of informal work with calculators in schools:

a) Work with first graders suggests the possibility that the standard display that feeds in digits from the right may lead to more number reversal errors in writing or reading numerals than displays (e.g., Hewlett Packard--HP) that feed in from the left. (That is, for 78, first 00000007., then 00000078 on most calculators, versus 7.0000000, then 78.000000 on some calculators.)

b) All displays obviously have limits on the number of digits displayable, and children somehow find that dismaying. That is, even though with eight places larger numbers can be displayed than children normally see or use, they want more. But there are obvious pedagogical possibilities in the lessons to be learned from such limitations, and in ways of coping with them (for example, with scientific notation--see below).

c) Children are dismayed by the penchant of calculators to give decimal answers where there "should" be whole numbers (e.g., (1/3) X 3 = 0.9999999). Some calculators (HP again) cleverly avoid such "difficulties" and presumably it would be possible to program a calculator to perform integer arithmetic correctly, or even to switch between integer and floating point modes. But it may be that the lessons of round-off error are too valuable in themselves to try to overcome the "wrongness" of the standard machine. How careful to be in this respect may be a function of the age of the child, but then again it may not be.

d) Again with respect to integer versus decimal arithmetic, most divisions give decimal answers, but there are many real life situations leading to division where quotient and remainder is the appropriate response, or where a remainder expressed as a fraction is easier to interpret than a decimal. (E.g., if I divide these 13 cookies among three children...). Remainders can be retrieved from the decimal answer and doing so may be a valuable lesson at some point, but perhaps not for children first learning about division. Calculators with a "Q" or"(Q,R)" option switch might be useful for early grades, first perhaps set by the teacher, later a choice to be made by the child according to the sort of problem at hand.

e) All the printed advice we've seen on buying school machines specifies algebraic logic, but that may be very wrongminded advice indeed. First of all, there is no indication that the equation format (e.g., 7 + 8 = ☐) for calculation is the natural way for children to think as opposed to the column format suggested by RPN logic (enter the numbers, then operate). Indeed some of our work with first graders suggests the contrary. Second, with equation format either parentheses are needed (more expensive machines increasingly have them on the keyboard) or intermediate results must be recorded (or stored in memory) and re-entered, or calculations must be rearranged in fairly ornate ways. There are useful learnings in doing these things, of course, but they must be balanced against the simplicity and lack of fuss of an RPN logic with automatic stacking and recall of intermediate results. Keeping track of what is in the stack (plus the capacity to verify that) also has valuable learning potential. It seems possible that RPN logic may be more easily transferable to thinking about computer programming. In other words, the choice is not nearly so clear-cut as has been suggested and there seems to us to be a rich field for pedagogical inquiry here. (The fact that essentially all inexpensive machines have algebraic logic may decide the issue without investigation, which would be unfortunate.)

f) We have already suggested that interesting pedagogical inquiries could follow from having machines available with more functions than are immediately familiar to a child. Might a third grader ask about square root? (If so, it could be easily explained, especially with a calculator at hand.) What about base-10 logarithms, say at a time when integer powers of ten and their link to our numerations system is already well in hand? (With a calculator and hence no nonsense about extrapolation from tables, etc., the discussion might be manageable much earlier than is now the case.) Would just the fact of seeing function names daily, even if never used or explained, make children more receptive later on to work with, for example, trigonometric functions? And so on.

g) In the typical calculator, all input and intermediate data is lost unless recorded by hand. Printing calculators keep track of all that, so are preferred by many for school use. But keeping a written record may be valuable in itself. It may also decrease the remoteness and mysteriousness of the answer machine. (We have long felt that keeping a written record provides non-trivial links between concrete experience and abstract number processing and something like that may carry over to calculators.) There is no doubt that children would have to be carefully taught to keep such records; every teacher is familiar with children who regularly erase all the work they do (or do it on a separate sheet to be thrown away), recording only the final answer. (Indeed, we find this in the majority of children, indicating that "neatness" etc. is taught as a virtue that takes precedence over clarity and demonstration of process.)

h) When decimal answers are involved, the standard eight-digit display often gives much more "accuracy" than is warranted. For younger children so many decimal places is often an overload of information. One can imagine pedagogical advantages to the ability to choose how many decimal digits will be displayed. If one can keep shifting the display (with automatic roundoff) for a given result (as with several of the HP calculators), then one can imagine a variety of beneficial learning exercises. Just the necessity to choose in advance how much information to keep would have benefits.

i) Scientific notation as available on many calculators might seem at first glance to be too complicated a code for use with young children (this code, in effect, changes every number to a pair of numbers with the first giving significant digits and the second a power of ten.) But our first reaction may be faulty and we should at least check on it by investigating when and how children can learn to use such a code with understanding. It may be that even primary children can use that code to some extent, and it seems very likely that many children could use it before it is taught in the present curriculum. Whenever it is teachable, it obviously opens up many new possibilities for using numbers in applications.

8. Are calculators durable enough for classroom use?

The history of classroom aids shows pretty clearly that things that get broken are simply no longer used. Teachers haven't the time or energy to get things repaired, even assuming there is money for repairs and they know where to send them.

With calculators, there are usually some initial failures with new machines--in one batch we bought, five of 24 machines had to be exchanged almost immediately but the failure rate in other batches was much lower. It seems to be the case that if they operate o.k. for a few hours, they are good for the long run. Even so, in handling about 50 calculators we have had to send four for repairs within the first year, even after initial exchanges. Dropping and other rough handling is infrequent--children seem to be pretty careful--and in any case we can trace no failures directly to such treatment. For inexpensive machines, repair costs after the usual one year warranty may exceed the price of a new machine.

From our experience it seems that if a school is to furnish machines, it should make sure of steady use for the first few hours to weed out and exchange initial failures; monitor carefully during the warranty period (usually one year), and then plan for about a 20% per year replacement (or repair) rate after the warranty period. We haven't had calculators long enough to know how long they will last before wearing out altogether.

9. Are losses from theft frequent, unmanageable?

This question has represented one of the major barriers to thinking about school use. One teacher flatly refused to try out our machines with the comment "Those things have legs!" The first batch of machines lent to several teachers quickly ended up in the school vault and only came out on our assurance that part of the trials were aimed at finding out if thefts would be unmanageable. Even so, on three occasions when a calculator from a batch was lost (by theft from desks or cabinets, not in use) teachers insisted on taking up a collection to reimburse us, and written feedback from one such class made it very clear that students caught hell from a teacher who donated to the collection.

Our experience in lending about fifty calculators in a wide variety of situations over about a one year period is that as far as we can tell none were stolen by the children actually using them. Five have been lost in schools from after-hours thefts from desk drawers on cabinets. Another five have been lost from loans to students and teachers from our somewhat loosely controlled mathematics/science curriculum laboratory at the University--which has the usual library problems of keeping things in circulation versus the risk of loss. In other words, with few special precautions our loss rate from school use and lab use has been about 10% for each. Lately we have etched numbers on calculators and made up boxes with numbered storage cells (so it is obvious when one is missing) and we have tightened up security arrangements; it is too early to tell the effect of this.

Our machines were lent when they were relatively expensive and relatively novel. With the same machines now less than $10 and with more and more in circulation, temptations may diminish. But our experience makes us cautious with respect to distribution of expensive special purpose machines. Also, we will now routinely require that students in our teacher training classes have their own calculators rather than lending calculators from the laboratory. The problems (to the students themselves) of loss, breakage, or extortion have made us think it unwise to issue calculators directly to children for out of school use, especially young children. Even with cheaper machines direct ownership (or rental) seems preferable to schools supplying calculators gratis.

10. How should calculators be powered?

Most of the printed advice to school buyers that we have seen advocates rechargeable calculators, but we aren't so sure. The extra cost is substantial. For a batch of machines kept in the classroom it is a major nuisance to keep them on chargers. Some rechargeable batteries are said to acquire a "memory" for undercharging--that is, if used before fully charged, they may not then take a full charge unless allowed to run completely down. If batteries do run down, a calculator can't be used until recharged, unless extra (and expensive) battery packs are available.

Although short life of (non-rechargeable) batteries was a major nuisance (and expense) with an early batch of machines we acquired, those we've gotten lately with alkaline 9v transistor batteries last for about 20 hours of actual use, which is good for several months for an individual who is careful about turning off the machine when not in use. On the other hand, a classroom set of calculators in continuous use during several class periods per day would use up the 20 hours in just a few days. A-C adaptors (not re-chargers) are available for battery operated calculators but few classrooms are set up with enough electrical plugs to make that practical.

Our experience is that problems in this area are a major deterrent to routine classroom use of calculators. On checking back with teachers to whom we loaned calculators we were frequently told that they and the kids loved them, but they were out of service from worn out batteries. (They were put back in service again only when I supplied the batteries.) Few classrooms have enough outlets to keep machines plugged in, and in any case that severely restricts portability. Batteries are expensive; teachers shouldn't buy them and schools may be unwilling to. The problem is unsolved and the most likely solution may be, again, individual student ownership of their own calculators, and hence responsibility to keep them in service. For school owned sets, and perhaps even for school use of student owned or rented calculators one solution might be rechargeable 9v transistor batteries (if these exist at modest cost), with straight exchange of run down for charged batteries, and special rechargers that handle many batteries at once. A teacher would then simply keep a supply of charged batteries, and turn in run down batteries for recharging in some central place.

Appendix B: Lessons Learned from Other Teaching and Learning Aids

The hand-held calculator is different from other teaching and learning aids which are used or have appeared in mathematics education. Experience with other aids may or may not generalize to calculators. But we feel it is helpful to examine other teaching aids for possible clues with respect to the future of calculators.

This particular essay was motivated by an earlier paper of Rogers on the same topic. She lists four features which seem to separate enduring teaching aids from the others:

> "1. It must be inexpensive and/or durable enough for child use.
> 2. It can be controlled by the learner.
> 3. It solves problems (or does things) that the learner wants done.
> 4. Its obsolescence is not contrived by manufacturers."

She concluded that "the electronic calculator has the potential, as assessed against these four criteria, to be a teaching aid of enduring value." (Rogers, 1974)

Blackboard

Rogers discussed the blackboard in detail, noting that it is a teaching aid extensively used in mathematics classrooms. It is an enduring teaching aid and, as she states, "it is almost unthinkable that a school would be built *without* lots of blackboards."

This is of course correct. But the blackboard example indicates to us that the second of Rogers' criteria needs to be modified. In our view, for those aids which tend to be used considerable control over when and how they are used remains with the *teacher*.

Let us compare the blackboard and calculator. In most classrooms, the blackboard is strictly off-limits to children unless the teacher gives permission. Once a student has a calculator, the teacher has lost some control, particularly since, as we have noted previously, there are not yet appropriate materials which a teacher could use to guide the student. In short, a minimum prerequisite for widespread use of calculators in the classroom would seem to be the availability of appropriate materials to allow the teacher a modicum of control over students' learnings. (There is a subtle distinction here: the materials themselves might be quite open-ended. What is important is that some structure be placed upon the learning environment.)

One must also note that the blackboard is a one-time expense usually covered in the cost of the original physical plant. Furthermore, a blackboard cannot be placed in a closet. It is constantly available. Few other teaching aids have these properties.

Slide Rule

The hand-held calculator has made the slide rule obsolete. For this reason, the two aids are often compared. The slide rule satisfies Rogers' criteria; also, its use can be controlled by the teacher either by the selection of problems or by using commercial materials designed for this purpose. From the above arguments it then stands to reason that, with available materials, the calculator would become a fixture in the classroom.

But slide rules have never received widespread acceptance in mathematics classrooms. Indeed, their use was banned in most mathematics classrooms, even though in later high school and college courses many students were using them concurrently in science courses.

Why have slide rules never found a place in the mathematics classroom? Most importantly, because they do not give exact answers or answers with as many significant figures as tables. There is a societal view of mathematics (also held by many mathematicians and educators) that the difference between mathematics and the sciences is that "mathematics is exact." In this paper we cannot examine that in detail, but it is important to note that this view has worked against those who advocate that estimation or probabilistic notions be given a more prominent role in the curriculum. Whether justified or not, paper and pencil instead of slide rule calculation has often been required in schools to achieve this supposed exactness.

While calculators give more precision (and do it more quickly) than slide rules, there are still limits on the number of significant figures. Also, relatively inexpensive calculators do not have as many features as their slide rule counterparts (logarithms, trig. functions, etc.). Calculators share the "trick" or "instant answer" aspect with slide rules.

In short, in the past most students below the college level have not used slide rules. Hence, if the calculator is seen only as a "better, faster slide rule," then its presence may not be felt in most school classrooms.

Computer

Although the hand-held calculator is the grandchild of the computer and they share a substantial theoretical base, the lesser cost and greater availability of the calculator puts it in a different ballpark with respect to the curriculum. Until the time that there is a terminal in every room or in a large number of homes, these two relatives must be treated as being from different families.

Other Mathematical Aids

There are other aids in mathematics: the overhead projector, abacus, geoboards, Cuisenaire rods, etc., but these do not seem to have enough in common with calculators to make analogies worthwhile. So we turn to aids in other disciplines.

Motion Pictures

Once used by only a few physical education departments, movies are now a mainstay of any major school athletic program. They are used to provide examples of what to do, what not to do, to go over performances, to help plan future strategies. As a result, the performance of teams has improved markedly in the past decade.

Movies are expensive to produce and show. But this example shows that, when a school feels that an aid will help performance, the school will go to the expense even when it is great and requires continual expenditures.

Typewriter

Typewriters and calculators have much in common. The typewriter makes it possible to write quickly, just as the calculator makes it possible to quickly compute. Just as there are times when a typewriter is not available when one wishes to write, so there will be times when a person wants to compute and a calculator will not be available. A typist does not lose writing skills and we can expect that an adult who has been taught to compute would not lose computation skills if a calculator were present. However, if a big writing job is desired, people tend to use typewriters or dictate for others to type. Similarly, we already see calculators in every place where a good deal of arithmetic needs to be done.

But how are typewriters used in schools? Only in a few schools are typewriters used to motivate reading and other language skills. (The SRA television commercial with 5th graders beaming about what typing has done for them comes to mind.) Even though most homes have typewriters, neither elementary nor high schools can require that work be typed. Rogers' first criterion (to be inexpensive) and the size and weight of a typical typewriter are the deterrents to greater usage.

Thus calculators and typewriters have corresponding features and we may learn from this. But the two major differences (size and cost) make it unwise to generalize all experiences with typewriters to calculators.

Dictionary

There are great similarities between dictionaries and calculators. Both aids satisfy each of Rogers' criteria. Both are available in models specifically designed for experts in a variety of fields and varying in sophistication. Costs are similar. Families have them at home and they are bought as gifts.

Similarities also exist with respect to the curriculum. Both aids are identified with one major subject area (dictionaries in Language Arts or English) but can be used elsewhere.

And similarities exist with respect to the content of their discipline. The dictionary makes almost all words in English equally accessible to its user, in much the same way that the calculator makes most real numbers equally accessible to the mathematics student. The dictionary theoretically makes spelling obsolete but you cannot make use of a dictionary unless you can spell a little; similarly, though the calculator theoretically makes it unnecessary to learn arithmetic skills, you need to have some of these skills in order to use the calculator efficiently.

These similarities make it reasonable to examine the use of dictionaries in classrooms. Some teachers make great use of the dictionary and plan units around it. The dictionary is a reference found in every elementary school, English classroom, and library. Curricula take into account that dictionaries exist but the richness of language arts seems to dictate that curricula are not designed around the dictionary.

It is entirely possible and quite reasonable that calculators will ultimately have the same type of usage in mathematics classrooms. This would imply that students will continue to learn basic mathematical skills (just as spelling is still taught) but not all computation will be known by students (just as few people can spell all the words they need).

In summary, if one looks at teaching and learning aids which have much in common with calculators and the extent to which these aids are used to their limit in classrooms, one is forced to predict that calculators will never be used to the extent suggested by some advocates. This would be in spite of the potential of the calculator to be a teaching aid of enduring value.

Reference

Joy Rogers, "The Electronic Calculator--Another Teaching Aid?" Unpublished paper, Loyola University of Chicago, 1974.

Appendix C: Are Calculators a Crutch?

In this section we consider a major argument against the use of calculators. It is that calculators are a crutch. This argument underlies the thinking represented in quotes like the following:

> "I understand the principle — get them motivated. But I have yet to be convinced that handing them a machine and teaching them how to push the button is the right approach. What do they do when the battery runs out? I see a lot of low-level math among college students who still don't understand multiplication and division. You take away their calculators and give them an exam in which they have to add 20 and 50, and they get it wrong. And I'm talking about business majors, the people who will soon be running my world."

(James R. McKinney, professor of mathematics, California Polytechnic State University of Pomona, as quoted in New York Times, Section IV, p. 7, Jan. 5, 1975)

The "crutch premise" is essentially that if you allow students to use a calculator for arithmetic problems which can be done by hand, then the students will be unable to do arithmetic when the calculator is absent. It is a corollary to this premise that calculators should not be used with young students who are still learning arithmetic. Another corollary to this premise is that calculators should not be used with older students who have not yet learned arithmetic — i.e., calculators should not be used in remedial classes.

We have discussed these notions before but now consider the crutch premise itself. We believe that if the premise is accepted, then the presence of calculators will have no effect upon the arithmetic curriculum. It is that strong a premise and we believe that it is a premise which is presently widely accepted. However we believe that the "crutch premise" is seriously open to question, both in its internal validity and in the validity of conclusions which are reached from it.

Let us give reasons for our belief. The crutch premise rests on a principle that a crutch is a bad thing. But in fact, for the injured person a crutch may be a good thing--even a necessity. In supermarkets and other stores, calculating cash registers are necessities because of their accuracy and speed. And these store calculators came into wide use at a time when the general populace was taught at least as much about calculation as it is today. For both the injured man and the supermarket, presumably, the "crutch" has become a "tool." But the capacity for a crutch (bad!) to be relabelled as a tool (good!) extends to many situations and the value judgments may be altered simply by which label is perceived as applicable.

It is common to cite the case of a real or theoretical student who takes a calculator into an exam only to have the battery run out, after which the student is helpless, confused, etc. We do not doubt the accuracy of such stories. But there are two questions to be asked: First, will the student allow this to happen on the next test? One would expect that a single experience of this kind would suffice and a similar thing would not recur. Second, for how many students in the same test was the calculator an asset? In short, the question may be whether to penalize the majority in a test because of unwise decisions which are bound to be made by a few.

When a computer or business machine breaks down in the real world, we know of few organizations which get rid of the idea of using the machine. Most get it quickly fixed, or they buy a new one. It is a necessity of life that machines break down or are unavailable but the increased level of performance with them more than makes up for these inevitable problems. Closer to the student's world, if an essay is required to be typed and the student's typewriter is not usable, the simple solution is to find another typewriter.

The "crutch" premise leads to the conclusion that calculators should not be used in remedial arithmetic classes (such as general mathematics courses in high schools or junior colleges). There is some reason to believe that this conclusion is false. Indeed, calculators may be more appropriate for these students than any other students of arithmetic.

Our reasoning is as follows. In the NAEP studies, 34% of 17-year-olds incorrectly answered 1/2 + 1/3; 26% incorrectly answered 1/2 X 1/4. These are substantial percentages and they occur despite the existence of special courses in arithmetic for high school students in which such skills are covered. In fact, it is now estimated that about 40% of entering freshmen are not sufficiently adept at arithmetic to enter algebra. These students have had 8 years of arithmetic and not had success at it. So we give them more, and the NAEP studies of 17-year-olds show that the total experience is still not successful. So at present such people are condemned never to have arithmetic to use in their lives. For such people the calculator is not a crutch but provides the only way to get a right answer. Furthermore, the more crucial issue is not how the calculation is accomplished but rather knowing when and how to use arithmetic to solve problems or answer questions that in fact matter in the lives of people. The ability to *use* calculation is what we should expect from those who have completed their study of arithmetic. And it is precisely this issue of skill vs. use that may be obscured by adamant insistence that *all* children must be excellent pencil and paper calculators.